The Standard Model

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"The Standard Model deserves more than 2 hours!"

Today: Structure of the SM
Wednesday: Your Questions!
- ↳ Higgs Mechanism?
- ↳ Electric Dipole Moments?
- ↳ Anomalous Magnetic Moments?
- ↳ Neutrino Masses?
- ↳ Proton Decay?
- ↳ Anything you want! (related to the SM, of course)

(I haven't yet prepared lecture 2, so I really mean it!)

My goal is to highlight SM features that are elegant and inevitable, as well as those things that are ugly and confusing.

Principles & Paradigms

✡ To the best of our knowledge, the Universe obeys the following principles:

- Quantum Mechanics
  - ↳ Unitarity
- Lorentz / Poincare Invariance
  - ↳ Mass & Spin
- CPT / Spin-Statistics
  - ↳ Integer spin ↔ Boson
  - ↳ Half-integer spin ↔ Fermion
- Locality / Causality
  - ↳ Free-field propagator (among other things)

⋮

- ?? Global Symmetries (?? ← Quantum Gravity)
  - ↳ Conservation Laws
- ?? Supersymmetry
  - ↳ Boson ↔ Fermion Partners
- ?? Naturalness
  - ↳ New Physics @ TeV scale

✡ Specifically in the SM, the following paradigms are realized:

- Spontaneous Symmetry Breaking
  ↳ Higgs Mechanism
- Confinement / Asymptopic Freedom
  ↳ Quantum Chromodynamics (QCD)
- Accidental Baryon / Lepton Number Conservation
  ↳ ?? Proton Stability

⋮

- ?? Neutrino Mass Generation
  ↳ Majorana vs. Dirac
- ?? Dark Matter
  ↳ Other lectures!
- ?? Baryogenesis
- ?? Unification
- ?? Inflation

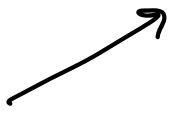
⋮

Principles have Consequences

Weakly-coupled $\Rightarrow$ Particle description

Lorentz symmetry $\Rightarrow$ Particles have well-defined mass

✡ Massive particles, go to their rest frame

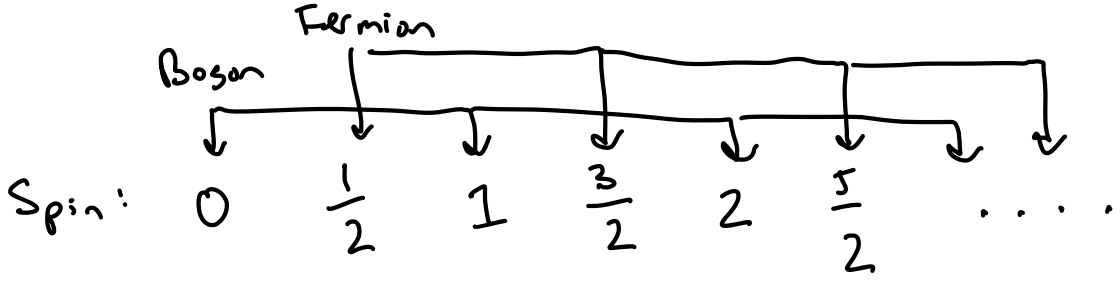


Boost $\Longrightarrow$ •

Rotations in 3-dimensions

$SO(3) \simeq SU(2)$

Spin!



$\Longrightarrow$ Can't be fundamental!

(For spin-1, Higgs mechanism

For spin-$\frac{3}{2}$, Super-Higgs mechanism)

- In Standard Model, only intrisically massive particle is spin-0 Higgs multiplet (4 real d.o.f)

✡ Massless particles, no rest frame

Rotations around axis

$SO(2) \simeq U(1)$

Helicity* ← ask me about CSEs

$\models$ Discontinuous massless limit

Helicity: $0 \quad \frac{1}{2} \quad 1 \quad \frac{3}{2} \quad 2 \quad \frac{5}{2} \quad \cdots\cdots$

$\models$ Can't have interactions

- In Standard Model:

  $3 \times 15 = 45$ spin $1/2$ "Weyl" fermions

  $1 + 3 + 8 = 12$ spin 1 bosons

- Remarkably, this information (plus 19 (or 20) free parameters) is all you need to describe Standard Model!

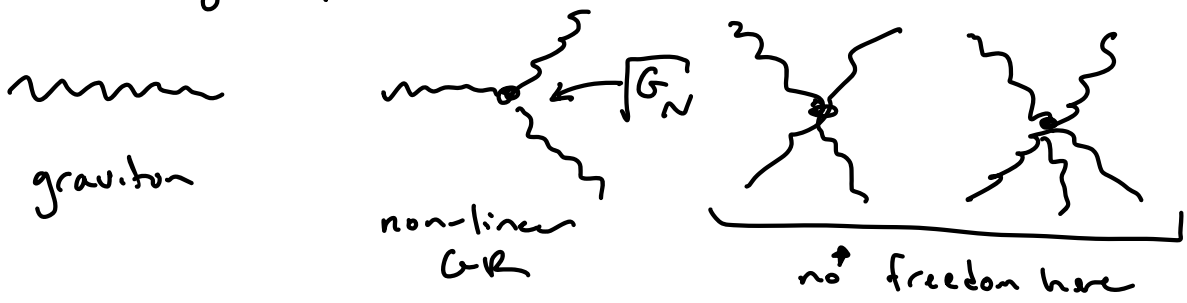
- Let's see why so much of the SM is inevitable.

Quantum Constraints on Massless Particles

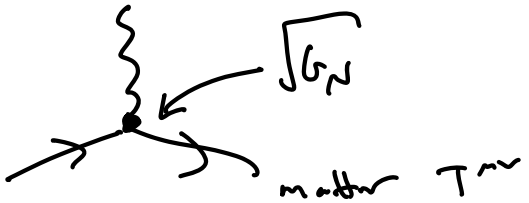
✡ Massless Spin-2 plus unitarity / locality / causality / ...

$\Rightarrow$ Einstein's gravity (!)

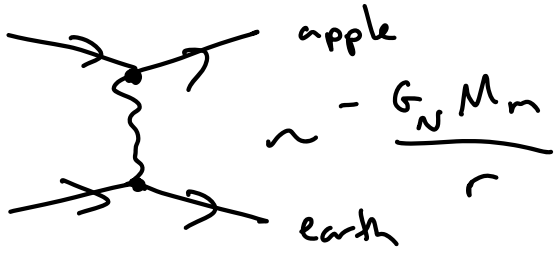
- Dictated by quantum mechanics!



- Very special form required!
- G_N is only free parameter!
- Must couple to conserved stress-energy tensor!



- Wild to think that Newton's law is a consequence of QM!



- Ask me what's the big deal about quantum gravity...

✡ Massless spin-$3/2$ plus unitarity | locality | causality | ...

$\Rightarrow$ Supergravity ("N" is number of gravitons)

- But doesn't seem (yet) to describe nature

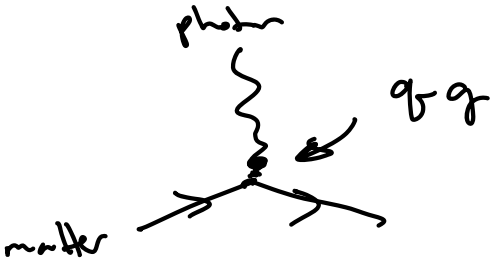
✡ Massless spin-1 plus unitarity | locality | causality | ...

$\Rightarrow$ Gauge theories!

- Just 1 massless spin-1? Maxwell's theory!

photon ~~~~ No self interactions allowed! ✡ kind of

- Must couple to conserved current!

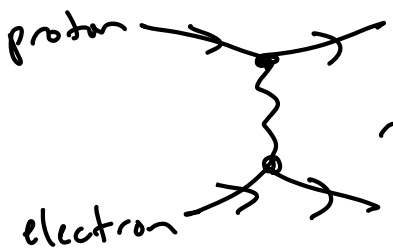


q = charge

g = coupling (free parameter)

- Wild to think that Coulomb's law is a consequence of QM!

- So much for beauty of local symmetries...



$\sim \frac{\alpha Q q}{r}$

- If you have more than 1 spin-1 boson, must have Yang-Mills theory!

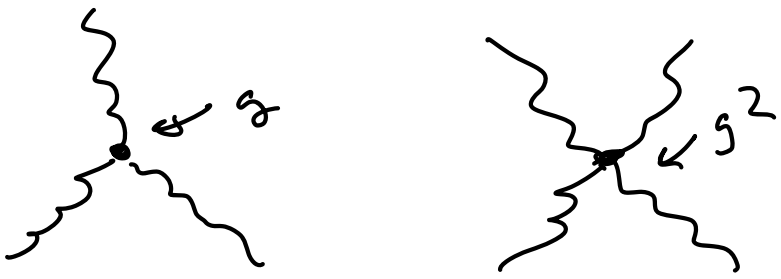
$$SU(2)_L \times U(1)_Y = 3 + 1 \text{ bosons}$$

for electroweak theory

$$SU(3)_C = 8 \text{ bosons}$$

for QCD

- Structure fixed by QM!



- Lots I can say about these theories since QCD is my specialty, but let's press on.

✡ Massless spin-$1/2$ fermions

$\Rightarrow$ No constraints (apart from above)

- Just need to specify charges under gauge groups.
- Weyl fermions = left-handed particle + right-handed antiparticle
- If you want a right-handed particle (with left-handed antiparticle), you need a separate Weyl fermion

✡ Massless spin-0 bosons

$\Rightarrow$ Radiatively unstable ✡ ← except Goldstone bosons which aren't fundamental

With this, we can build SM on one page!

But let's warm up with QED

Building Quantum Electrodynamics

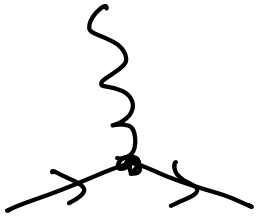
- Massless Spin-2 graviton (G_N)
- Massless spin-1 photon (α_{EM})
- 2 spin-$1/2$ Weyl fermions $\Rightarrow$ electron

	$U(1)_{EM}$ charge
e	-1
e^c	$+1$

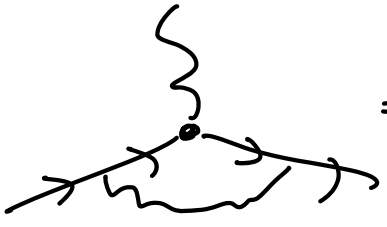
"c" is a name

pair up to make massive electron

- electron $\overset{\text{antiparticle}}{\longleftrightarrow}$ positron



$\Rightarrow$ gives electric charge & magnetic coupling $(g=2)$



$\Rightarrow$ gives running coupling & anomalous $(g-2)_e$

- Conserves C, P, & T, so no electric dipole moment

Lecture 2

- Thanks for your awesome questions! Keep them coming!
- Last time:

  Massless spin-2 $\Rightarrow$ GR

  Massless spin-1 $\Rightarrow$ YM

  Spin-$\frac{1}{2}$, spin-0 $\Rightarrow$ Unconstrained-ish

- The above is consequence of deep principles
- With this, we built QED
- Today:
  - Build the Standard Model
  - The Higgs mechanism
  - The SM as an EFT
- Cutting Room Floor (Sorry!):
  - Strong CP, EDMs, Flavor Physics, Anomaly Cancelation, RG Flow, Confinement, Unification, ...

Building the Standard Model

- Massless spin-2 graviton (G_N)
- 12 massless spin-1 bosons $(\alpha_Y, \alpha_W, \alpha_S)$
- 3 copies of 15 spin-$1/2$ Weyl fermions

	$U(1)_Y$	$SU(2)_L$	$SU(3)_C$
e^c	$+1$	—	—
$l = \begin{pmatrix} e \\ \nu \end{pmatrix}$	$-\frac{1}{2}$	2	—
u^c	$-\frac{2}{3}$	—	$\overline{3}$
d^c	$+\frac{1}{3}$	—	$\overline{3}$
$q = \begin{pmatrix} u \\ d \end{pmatrix}$	$+\frac{1}{6}$	2	3

- First observation: no fermion masses allowed!

- At this point, just 3 (or 4) parameters
- Where are the other 16 parameters??
  - ⇒ 9 fermion masses (from where?)
  - ⇒ 3 mixing angles (between what??)
  - ⇒ 2 CP-violating phases (huh???)
- Still 2 parameters to go...
  - ⇒ 1 boson mass
  - ⇒ 1 boson vacuum expectation value
- What boson is this?!
  - ⇒ Higgs boson! (spin-0)

	$U(1)_Y$	$SU(2)_L$	$SU(3)_C$
h	$-\frac{1}{2}$	2	—
h^+	$+\frac{1}{2}$	$\overline{2} \equiv 2$	—

(complex conjugate)

- Most SM complications arise from these 4 (real) bosons!

Where are we now?

- We have the ingredients, but not the implications
- "The SM deserves more than 2 hours"
- Let's start with vacuum structure

$$\langle V \rangle \sim \Lambda^4 \longleftarrow \text{cosmological constant}$$

$$\langle h^\dagger h \rangle \sim v_{EW}^2 \longleftarrow \text{electroweak symmetry breaking}$$

$$\langle d d^c \rangle \sim \langle u u^c \rangle \sim \Lambda_{QCD}^3 \longleftarrow \text{QCD confinement / chiral symmetry breaking}$$

- $M_{Pl} >> v_{EW} \gtrsim \Lambda_{QCD} >> \Lambda$

Hierarchy problem ☹

Dimensional transmutation ☺

Cosmological constant problem 😕

- The Vacuum is dynamic and confusing!

The Higgs Mechanism

- $\langle L^\dagger h \rangle \sim v_{EW}^2 \implies \langle h \rangle = \begin{pmatrix} 0 \\ v_{EW} \end{pmatrix}$
- But h is charged under $SU(2)_L \times U(1)_Y$!
- Higgs picks out preferred direction

$$\implies Q_{EM} = Y + T_3 \quad \text{unbroken}$$

- (Massless) spin-1 bosons (2 helicities each)

$$Y^0 \quad W^0 \quad W^+ \quad W^-$$

- Spin-0 bosons (1 mode each)

$$h^0 \quad \phi^0 \quad \phi^+ \quad \phi^-$$

- When the dust is settled ...

photon (γ): mix of Y^0 & W^0] massless spin-1

Z boson: mix of Y^0 & W^0 plus ϕ^0] massive spin-1

$W^\pm$ boson: $W^\pm$ plus $\phi^\pm$] complex massive spin-1

Higgs boson: h^0] real massive spin-0

Below the Electroweak Scale

- The principles of QM still hold, but the ingredients are changing!
- Massless spin-2 graviton
- 9 massless spin-1 bosons
  photon plus 8 gluons
  $U(1)_{EM}$ × $SU(3)_C$
- Still 3×15 Weyl fermions (ignoring top quark subtlety)

	$U(1)_{EM}$	$SU(3)_C$	
e	-1	—	Dirac mass
e^c	$+1$	—	
ν	0	—	?? Majorana mass
u	$+2/3$	3	Dirac mass
u^c	$-2/3$	$\overline{3}$	
d	$-1/3$	3	Dirac mass
d^c	$+2/3$	$\overline{3}$	

Eventually, $SU(3)_C$ confines,

quarks form baryons / mesons,

baryons form nuclei, bind with

electrons to make atoms,

atoms combine to make molecules

molecules do their chemistry, etc...

Lesson: • Complexity of SM mostly
associated with spin-0
Higgs boson

• Elegance of SM mostly
associated with massless
spin-1 (and spin-2) structure

The SM as an Effective Field Theory

- My view: Everything consistent with core principles should be assumed to occur unless you can show (experimentally or theoretically) that it does not.

- The expected size of effects should follow from effective field theory scaling. If the observed size does not, then you are probably missing a principle / paradigm.

- E.g. top quark mass: $(\lambda_t q h^+ t^c)$

  $m_t\, t t^c$ breaks electroweak symmetry, so

  m_t should be proportional to v_{EW}

  m_t: 173 GeV — v_{EW}: 246 GeV

  Works!

- E.g. electron mass: $(\lambda_e l h e^c)$

  $m_e\, e e^c$ breaks EW symmetry, so

  m_e should be proportional to v_{EW}

  m_e: 0.000511 GeV — hmm...

- E.g. Majorana neutrino mass: $(lh\,lh)$ ↑↑

$m_\nu\, \nu\nu$ breaks EW symmetry twice!

m_ν should be proportional to v_{EW}^2

$$m_\nu^{est} \sim \frac{v_{EW}^2}{\Lambda_\nu}$$ ← some new scale!

Maybe Λ_ν is M_{Pl}? (10^{18} GeV) $\Rightarrow m_\nu^{est} \sim 10^{-13}$ GeV

$m_\nu^{obs} \sim 10^{-10}$ GeV surprisingly close!

So we expect, naively, some kind of new physics at Λ_ν. In this telling, Majorana neutrino masses expected, while Dirac neutrino mass $(m_\nu^{Dirac}\, \nu\, \nu^c)$ ↑ $\ll v_{EW}$ would be weird.

- E.g. Neutron Electric Dipole Moment

$$d_N^{est} \sim \Lambda_{QCD}^{-1}$$ ↑ GeV since QCD has CP-violating θ angle

$$d_N^{obs} \sim 10^{-10}\, \Lambda_{QCD}^{-1}$$ hmm...

- Last example: proton decay.

Nothing* forbids $u^c u^c d^c e^c$ interaction

(For $SU(3)$, $\bar{3} \otimes \bar{3} \otimes \bar{3} = 1 \oplus \cdots$)

Violates baryon number and lepton number, but absence of global symmetries is not a principle (in fact it might be requirement of quantum gravity!)

This is a "dimension-6" operator, so amplitude is proportional to Λ_p^{-2}

$$\tau_{\text{proton}}^{\text{est}} \sim \frac{\Lambda_p^4}{\Lambda_{QCD}^5}$$

Λ_p^4 ← squared amplitude; Λ_p ← maybe this is M_{Pl}?; Λ_{QCD} ← only other scale in problem

$$\sim 10^{72}\ \text{GeV}^{-1} \sim 10^{40}\ \text{years}$$

$$\tau_{\text{proton}}^{\text{obs}} > 10^{34}\ \text{years}$$

Surprisingly close! I view this as a "prediction" of SM.

Concluding Thoughts

- "The Standard Model deserves more than 2 hours"
- I hope I have at least given you a glimpse into the richness of this theory.
- Inevitability vs. Confusion
  - ⇓ massless spin-2 & spin-1
  - ⇓ massive spin-0 Higgs boson
- Principles vs. Paradigms
  - Which is pathway to enlightenment?
  - My view: No evidence that core principles are broken, but we don't know what paradigms are realized in our universe
  - But you do you! Listen to your elders... ... and then ignore/upstage them.
- Theory ⟷ Experiment
  - Both are essential to understand our universe
  - Make friends!