Gravity tests of all scales

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School on Table-Top Experiments for Fundamental Physics

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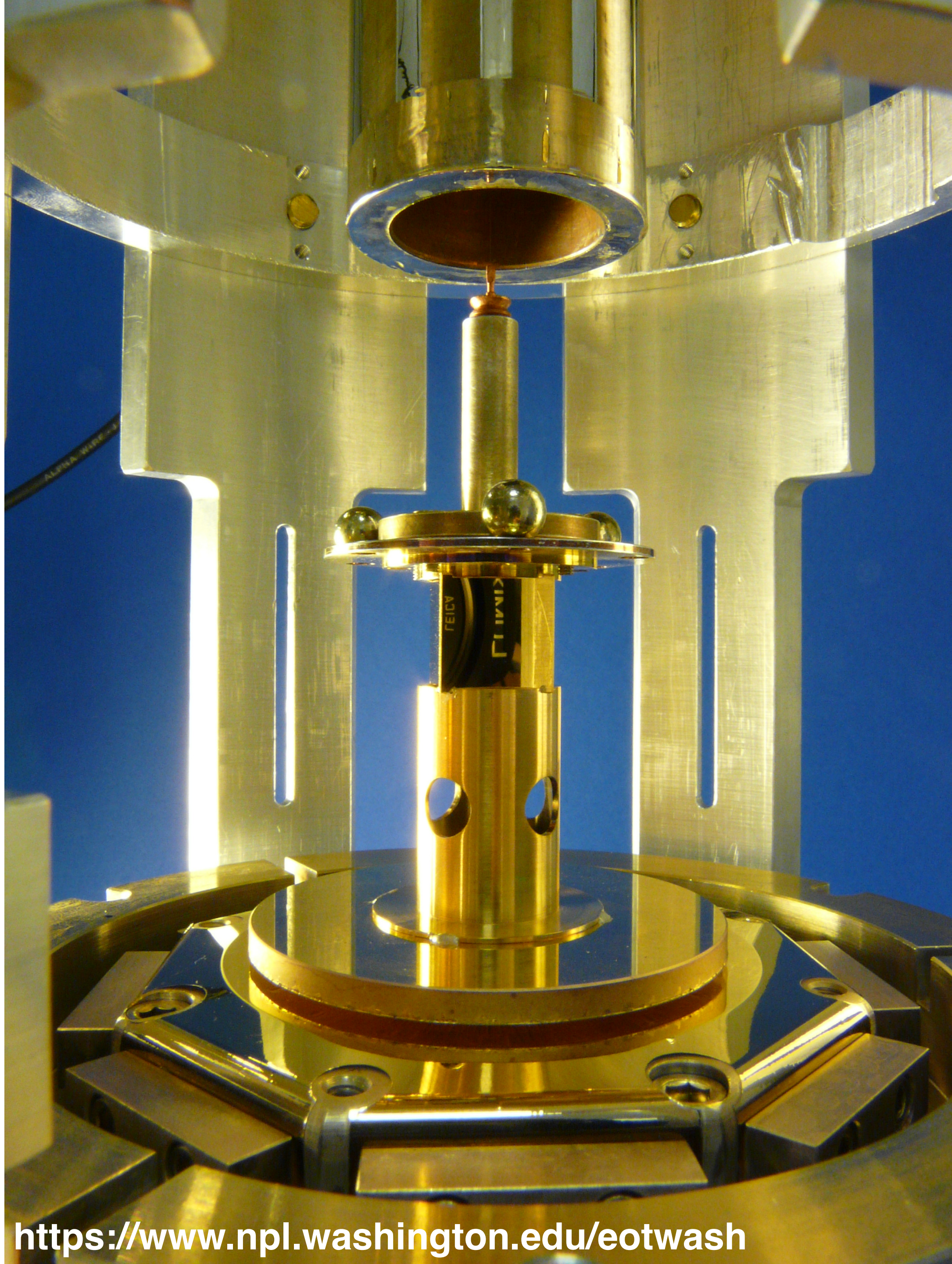
PERIMETER INSTITUTE



Plan for these talks:

Lecture 1 (today):

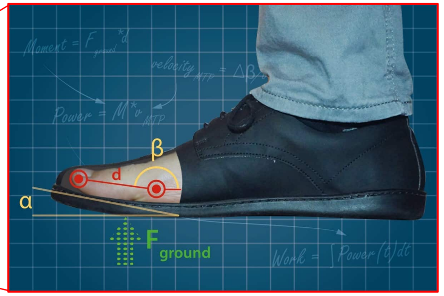
Overview of best current (lab) experiments and constraints

Lecture 2 (tomorrow):

New experimental techniques and frontiers in the coming years

Gravity is “weird”

- Despite the fact that we’ve been studying gravity longer than any other force, it is in some sense the least understood of the fundamental forces
- It is “weird” in two ways:
  1. **It is incredibly weak**



Just a “few” electrons can hold us against the entire mass of the earth

M_{earth} = 6 x 10^{24} kg (10^{51} GeV!!)

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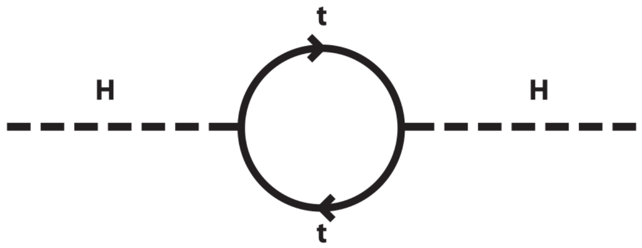
     → “Gauge Hierarchy Problem”

gravity ↑

$$\frac{\alpha_{grav}}{\alpha_{EM}} \sim \frac{G m_p^2}{e^2} = 10^{-38} \qquad (\hbar = c = 1)$$

Newton’s constant (G), Proton mass (m_p), Electron charge (e)

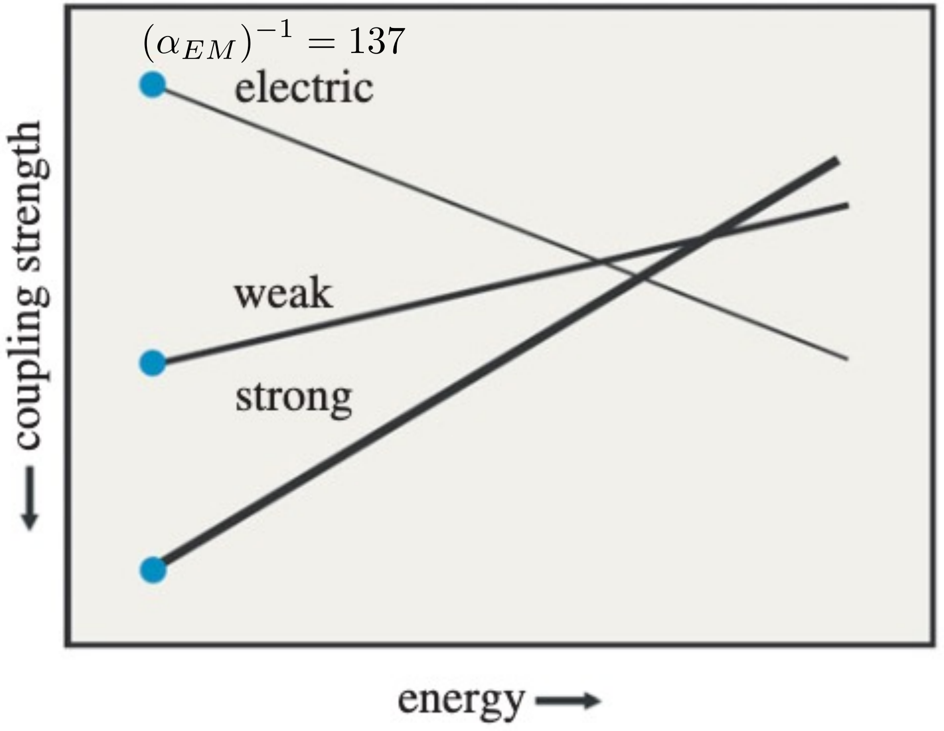
Why is the Higgs mass so much lighter than the Planck mass?



$$\Delta m_{\rm H}^2 = -\frac{|\lambda_f|^2}{8\pi^2}[\Lambda_{\rm UV}^2 + \ldots].$$

Quantum corrections naively depend on the cutoff (e.g. Planck Scale)

https://en.wikipedia.org/wiki/Hierarchy_problem

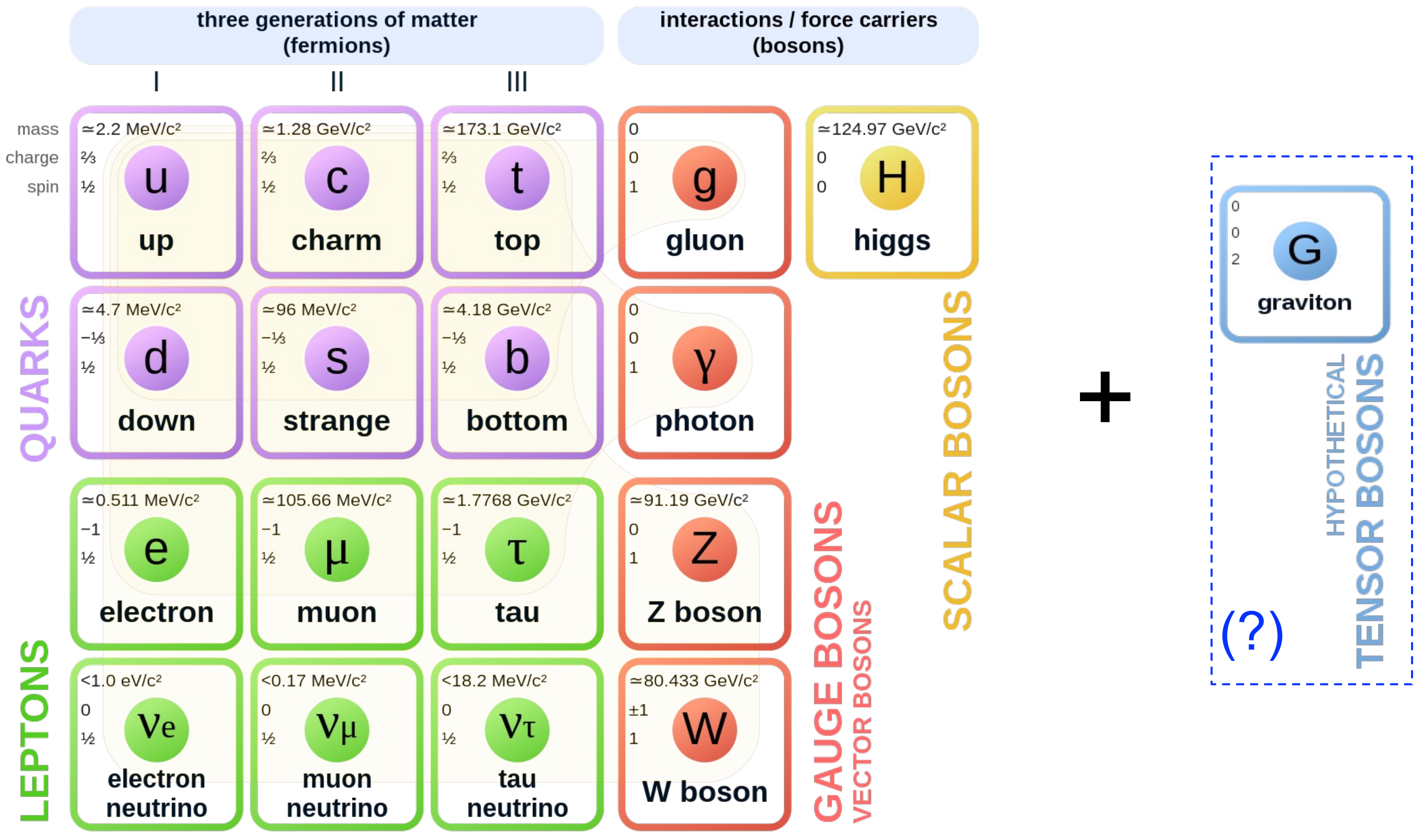


F. Wilczek, https://doi.org/10.1098/rsta.2015.0257

Gravity is "weird"

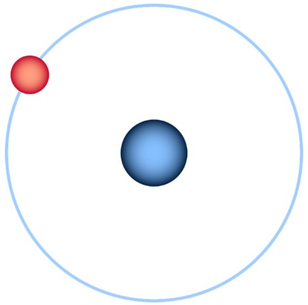
- Despite the fact that we've been studying gravity longer than any other force, it is in some sense the least understood of the fundamental forces
- It is "weird" in two ways:
  2. **We don't have a consistent microscopic theory (GR + QM = ?)**

Standard Model of Elementary Particles



Why are gravitational experiments hard?

- The reason we don't have understand gravity at microscopic distances is related to the first issue
  - This is also what makes any real lab experiments (not using astrophysical masses) so hard!



In SI units:

$$V_{EM} \sim 10 \text{ eV}$$

$$V_{grav} \sim 10^{-38} \text{ eV}$$

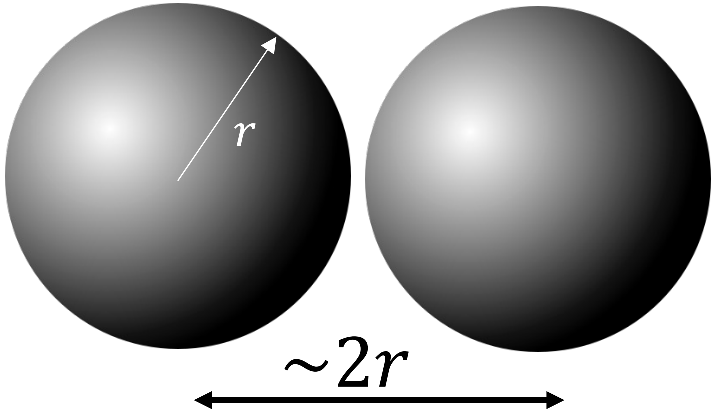
Extremely tiny effect in everyday experiments with quantum systems!

Planck mass/energy:

$$\sqrt{\frac{\hbar c^5}{G}} \sim 10^{19} \text{ GeV}$$

Not possible to directly probe the relevant energy scales at colliders!

In general for lab experiments:



$$F_{grav} \sim G\rho^2 r^4$$

$$a_{grav} \sim G\rho r$$

Upper limit on
$\rho \sim 20$ g/cm^3

At the same time, electrical non-neutrality of real objects grows in importance!

Practically this means:

- Experiments with ~cm scale masses (~nN) are doable
- 0.1 - 1 mm scale masses are smallest gravity has been measured for (~fN)
- 1-10 um scale (~zN) might be an ambitious future limit (Heisengberg uncertainty, shielding Casimir backgrounds, etc)

What can we test (in the lab)?

- Laboratory densities and masses are always in the Newtonian limit (no strong field general relativity in the lab!), i.e.:

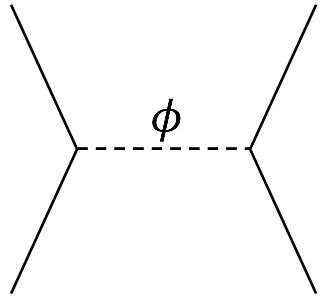
$$\frac{GM}{rc^2} \ll 1 \quad \text{and} \quad v \ll c$$

- In this case, Newton's law is an excellent approximation to GR: $$V = \frac{Gm_1m_2}{r}$$

- Can basically measure 3 things:

  1. Does the Newtonian $1/r^2$ dependence hold at all length scales?
  2. Is the force independent of the composition of the masses?
  3. What is the exact value of G?

Generic toy model in which these can fail:



Scalar ϕ coupling to neutrons gives 5th force in non-rel. limit:

$$V_{5th} = \frac{g_n^2}{4\pi}\frac{e^{-m_\phi r}}{r} \qquad (\hbar = c = 1)$$

Experimentally constrain:

Is the same regardless of test mass composition?

$$V = \frac{Gm_1m_2}{r}\left(1 + \alpha e^{-r/\lambda}\right)$$

"Yukawa Potential"

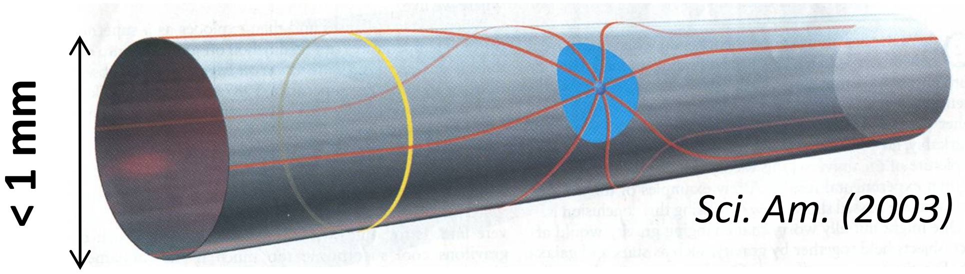
Difficult, systematics limited measurements

5th forces apparent at short distances?

Additional possibilities

- Instead of adding a "5th force," modify gravity itself:
  - E.g., $1/r^2$ law arises from Gauss's law in 3D, but would differ with more spatial dimensions!
  - This provides a possible solution to the Gauge Hierarchy problem
  - But, since the initial proposal:
    - Eot-Wash (torsion balance) has measured Newton's law down to ~50 μm
    - No evidence for signatures at the LHC (TeV scale)

Large extra dimensions:



e.g., Arkani-Hamed et al., Phys. Lett. B 429, 263 (1998)

- Add cosmologically relevant 5th forces (motivated by dark energy) that evade solar system and laboratory tests
  - Generally require a "screening mechanism" that turns force off near mass
    - Examples:
      - Chameleons (range of the force depends on local mass density)
      - Symmetrons (coupling depends on local mass density)
      - Vainshtein mechanism (self-couplings boost kinetic terms in vicinity of mass)

"Dark energy length scale" (possibly just numerology!)

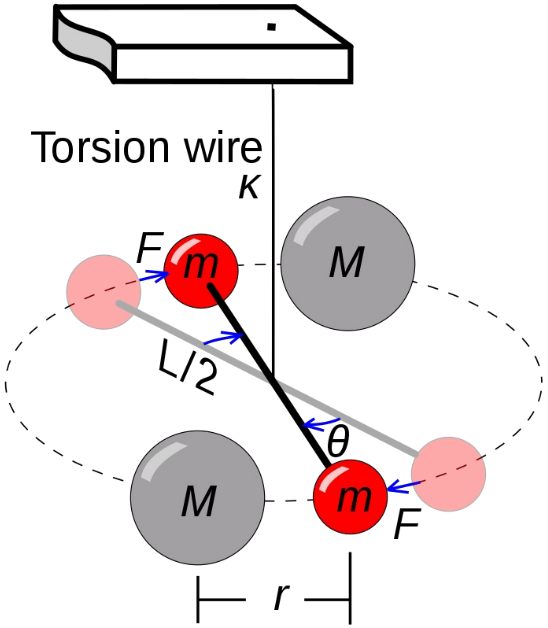
$\Lambda \sim 2$ meV

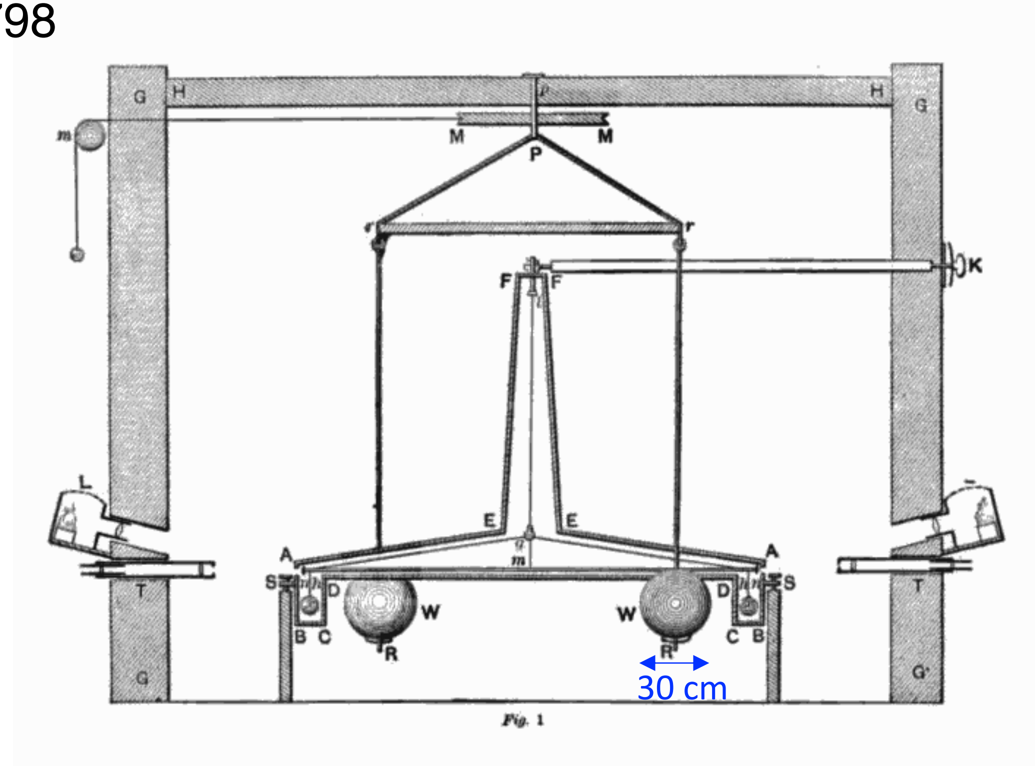
$$\frac{\hbar c}{\Lambda} \sim 80\ \mu\mathrm{m}$$

"Beyond the Cosmological standard model" Joyce, Jain, Khoury and Trodden, arXiv:1407.0059

Overview of experimental techniques

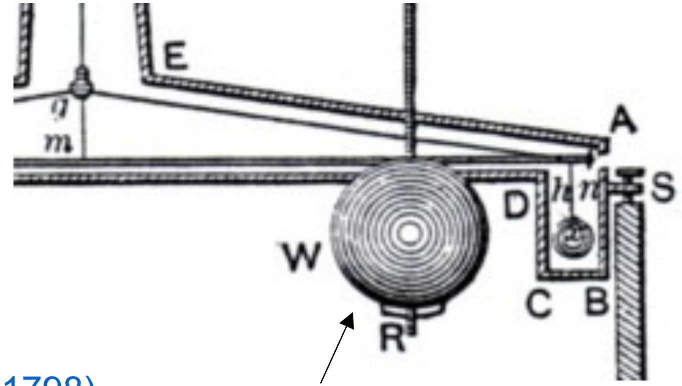
- Due to its extreme weakness, gravity is difficult to study as described previously
- For large enough masses, gravitational effects can add up while E&M effects cancel (positive/negative charges)
- "Standard technique" is a torsion balance:
    - First designed/built for gravity by John Michell (independent of Coulomb), Cavendish refurbished and presented results in 1798





[Cavendish, H., *Phil. Trans. Royal Soc. London*, **88** 469 (1798)]

In modern interpretation, measured G to ~1% accuracy!

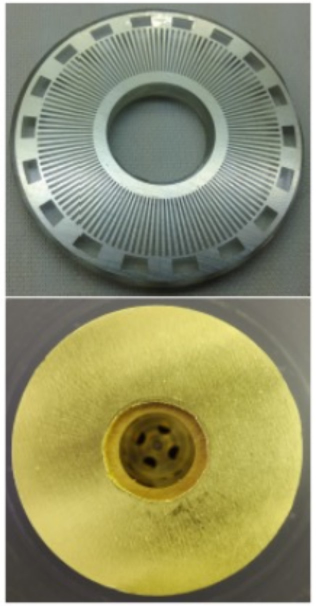


160 kg Pb attractors

Modern torsion balances

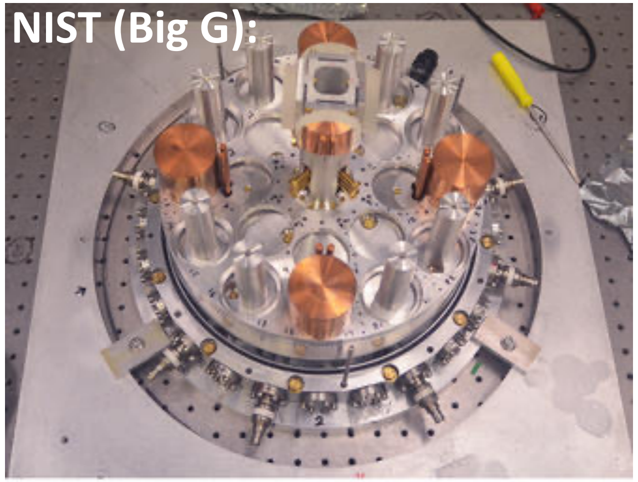
- Torsion balances remain the most sensitive current method for measuring gravity in the lab over ~10 um to meter length scales

Eot-wash (short-distance tests):

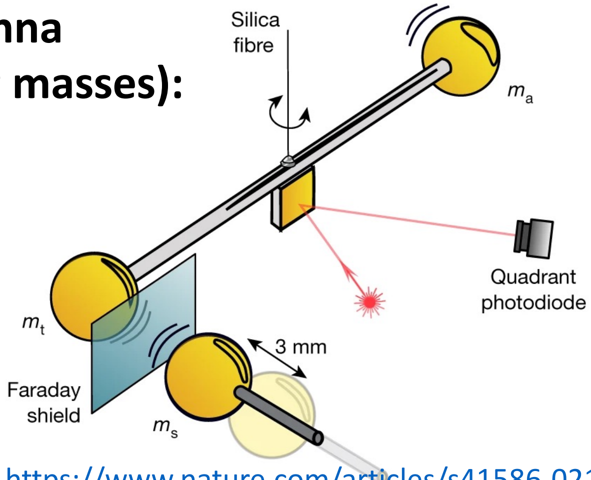
https://doi.org/10.1103/PhysRevLett.124.101101

NIST (Big G):



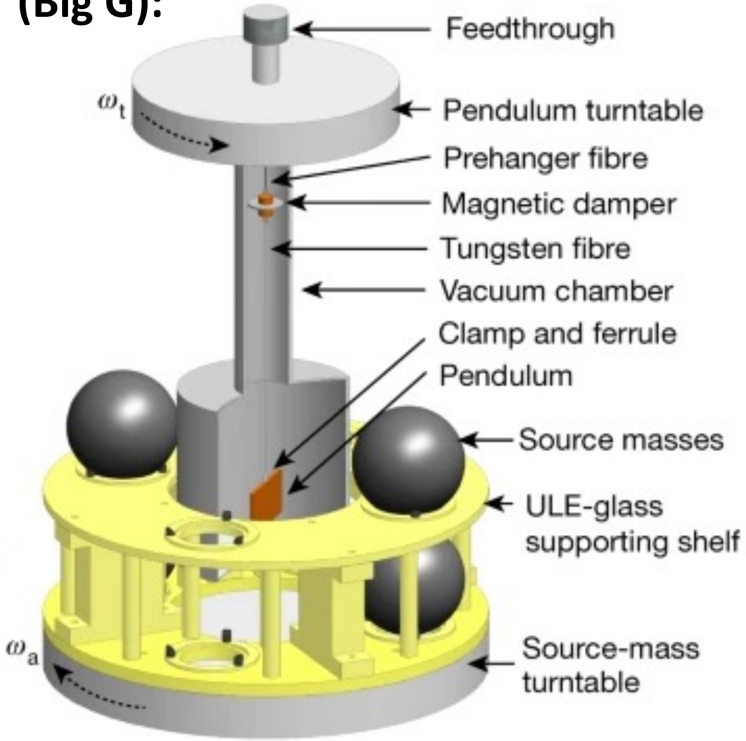
https://www.nist.gov/news-events/news/2016/11/big-g-redux-solving-mystery-perplexing-result

Vienna (mg masses):



https://www.nature.com/articles/s41586-021-03250-7

HUST (Big G):

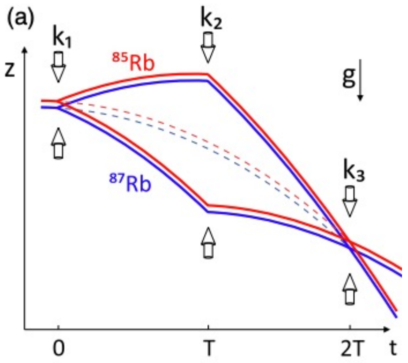


https://www.nature.com/articles/s41586-018-0431-5

Beyond torsion balances

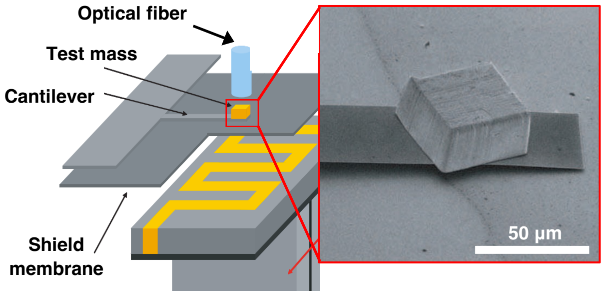
- To push to shorter length scales, or higher precision, a number of new techniques have been developed beyond torsion balances

Atom interferometry:

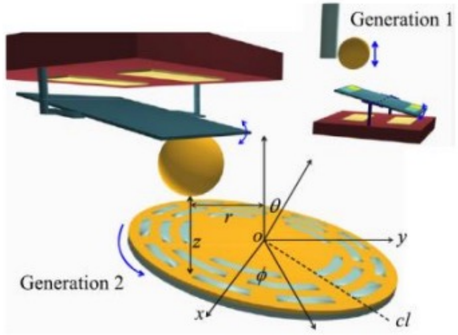


https://journals.aps.org/prl/abstract/10.1103/PhysRevLett.125.191101

Nano/micro oscillators:

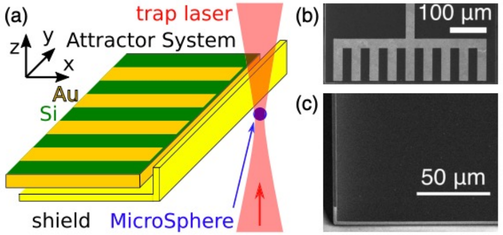


https://journals.aps.org/prd/abstract/10.1103/PhysRevD.78.022002

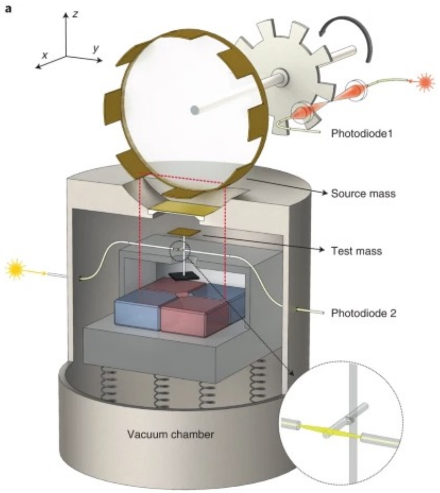


https://journals.aps.org/prl/abstract/10.1103/PhysRevLett.116.221102

Levitated systems:

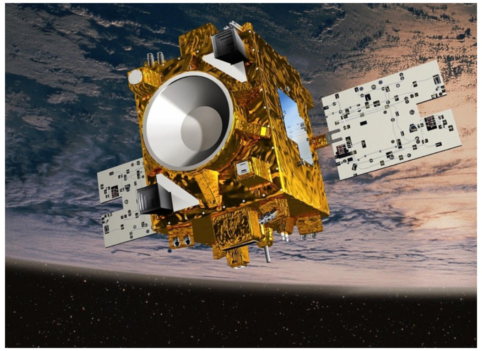


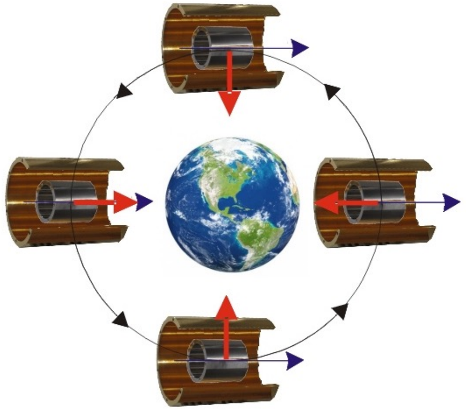
https://journals.aps.org/prd/abstract/10.1103/PhysRevD.104.L061101



https://www.nature.com/articles/s41567-022-01706-9

Space based tests:

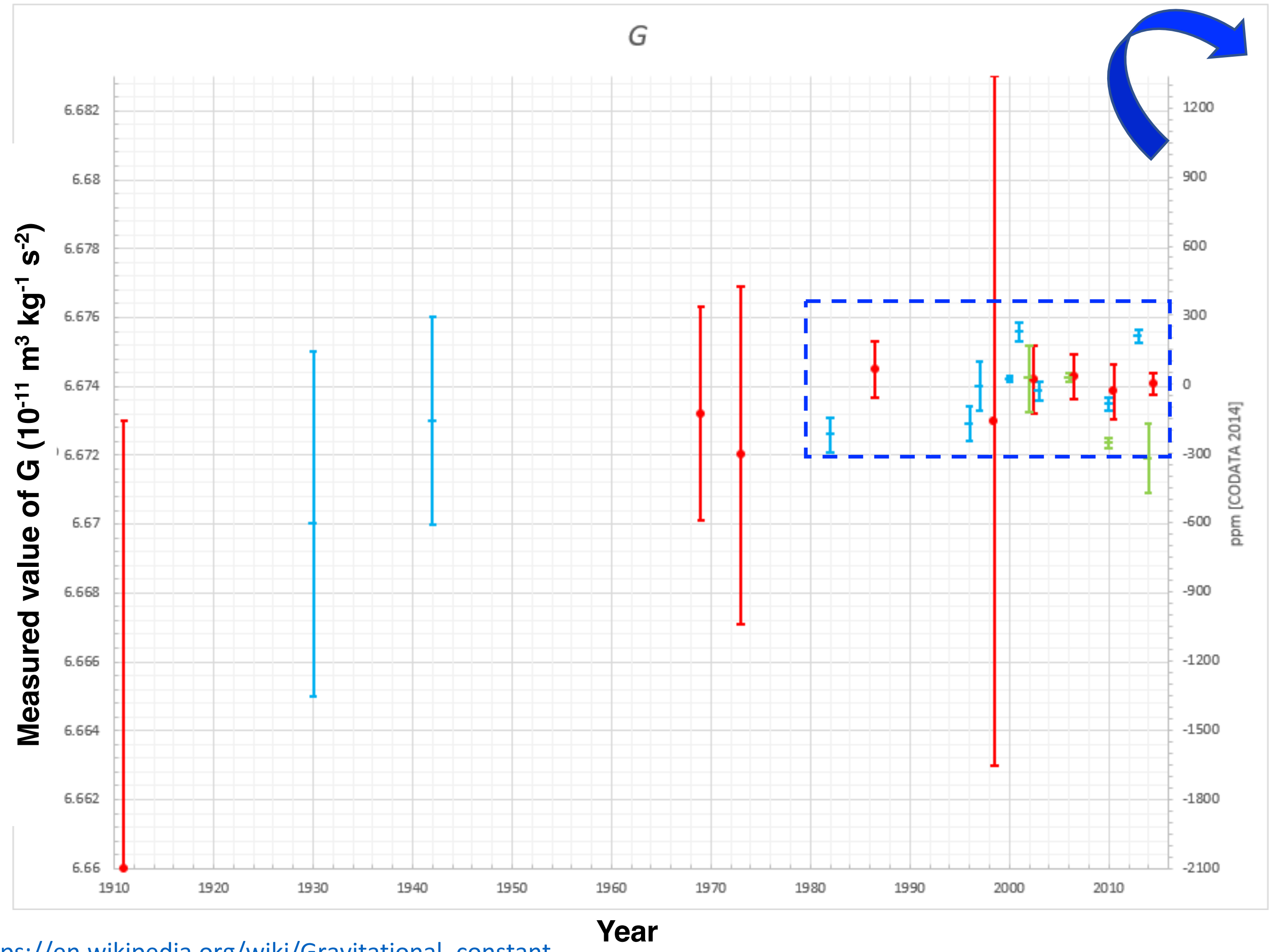




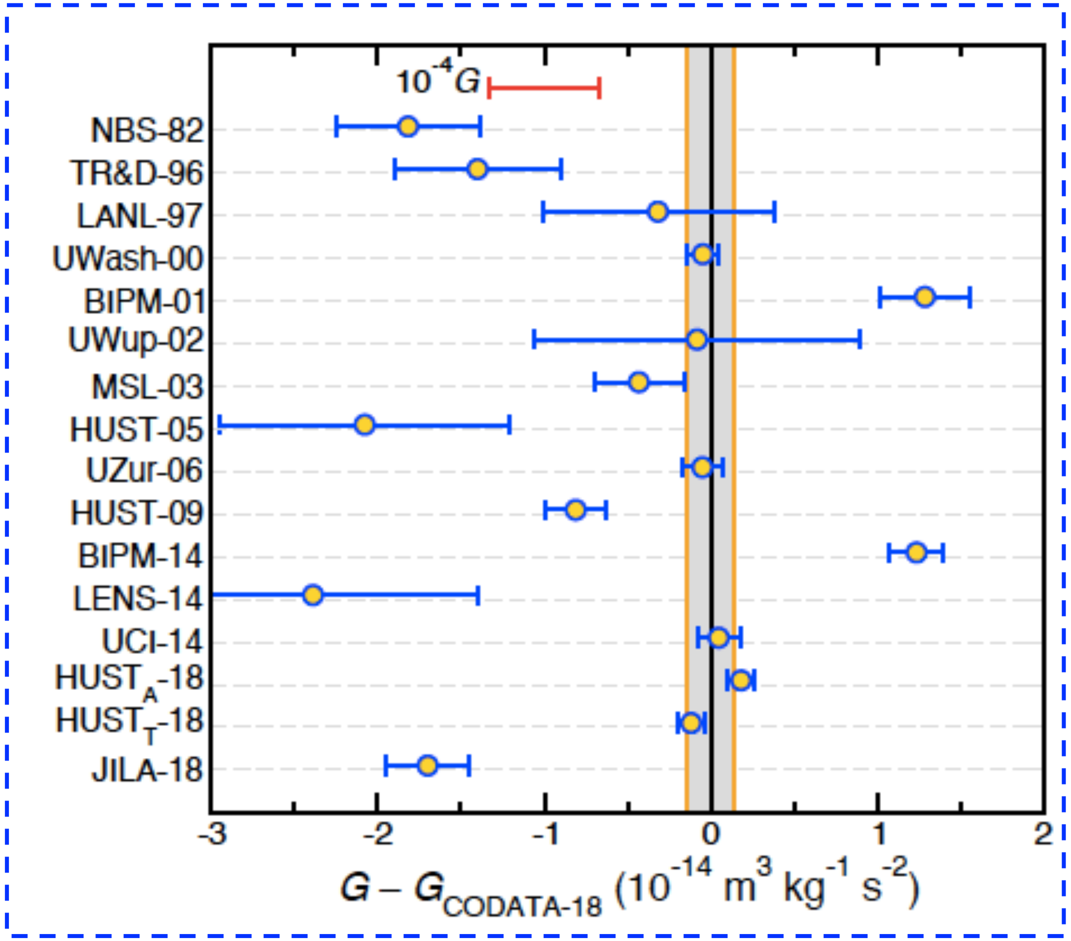
https://journals.aps.org/prl/abstract/10.1103/PhysRevLett.129.121102

Historical measurements (“Big G”)

- Measurements of overall strength of G are extremely challenging:
  - Systematics limited: (E&M effects, knowledge of source mass distribution, vibrations, Newtonian noise, …)



https://en.wikipedia.org/wiki/Gravitational_constant



https://www.nist.gov/image/big-gpng

Historical measurements ("Big G")

- Relative uncertainty for the coupling measured for gravity is ~10^5 times larger than for E&M:

Particle Data Group summary of physical constants:

Quantity	Symbol, equation	Value	Uncertainty (ppb)
speed of light in vacuum	c	$299\ 792\ 458$ m s^{-1}	exact
Planck constant	h	$6.626\ 070\ 15 \times 10^{-34}$ J s (or J/Hz) §	exact
Planck constant, reduced	$\hbar \equiv h/2\pi$	$1.054\ 571\ 817\ldots \times 10^{-34}$ J s	exact*
		$= 6.582\ 119\ 569\ldots \times 10^{-22}$ MeV s	exact*
electron charge magnitude	e	$1.602\ 176\ 634 \times 10^{-19}$ C	exact
conversion constant	$\hbar c$	$197.326\ 980\ 4\ldots$ MeV fm	exact*
conversion constant	$(\hbar c)^2$	$0.389\ 379\ 372\ 1\ldots$ GeV^2 mbarn	exact*
electron mass	m_e	$0.510\ 998\ 950\ 00(15)$ MeV/c^2 $= 9.109\ 383\ 7015(28) \times 10^{-31}$ kg	0.30
proton mass	m_p	$938.272\ 088\ 16(29)$ MeV/c^2 $= 1.672\ 621\ 923\ 69(51) \times 10^{-27}$ kg	0.31
		$= 1.007\ 276\ 466\ 621(53)$ u $= 1836.152\ 673\ 43(11)\ m_e$	0.053, 0.060
neutron mass	m_n	$939.565\ 420\ 52(54)$ MeV/c^2 $= 1.008\ 664\ 915\ 95(49)$ u	0.57, 0.48
deuteron mass	m_d	$1875.612\ 942\ 57(57)$ MeV/c^2	0.30
unified atomic mass unit**	$u = (\text{mass } ^{12}\text{C atom})/12$	$931.494\ 102\ 42(28)$ MeV/c^2 $= 1.660\ 539\ 066\ 60(50) \times 10^{-27}$ kg	0.30
permittivity of free space	$\epsilon_0 = 1/\mu_0 c^2$	$8.854\ 187\ 8128(13) \times 10^{-12}$ F m^{-1}	0.15
permeability of free space	$\mu_0/(4\pi \times 10^{-7})$	$1.000\ 000\ 000\ 55(15)$ N A^{-2}	0.15
fine-structure constant	$\alpha = e^2/4\pi\epsilon_0\hbar c$	$7.297\ 352\ 5693(11) \times 10^{-3} = 1/137.035\ 999\ 084(21)$† ‡‡	0.15
classical electron radius	$r_e = e^2/4\pi\epsilon_0 m_e c^2$	$2.817\ 940\ 3262(13) \times 10^{-15}$ m	0.45
(e^- Compton wavelength)/2π	$\lambdabar_e = \hbar/m_e c = r_e \alpha^{-1}$	$3.861\ 592\ 6796(12) \times 10^{-13}$ m	0.30
Bohr radius ($m_{\text{nucleus}} = \infty$)	$a_\infty = 4\pi\epsilon_0\hbar^2/m_e e^2 = r_e \alpha^{-2}$	$0.529\ 177\ 210\ 903(80) \times 10^{-10}$ m	0.15
wavelength of 1 eV/c particle	$hc/(1\text{ eV})$	$1.239\ 841\ 984\ldots \times 10^{-6}$ m	exact*
Rydberg energy	$hcR_\infty = m_e e^4/2(4\pi\epsilon_0)^2\hbar^2 = m_e c^2\alpha^2/2$	$13.605\ 693\ 122\ 994(26)$ eV	1.9×10^{-3}
Thomson cross section	$\sigma_T = 8\pi r_e^2/3$	$0.665\ 245\ 873\ 21(60)$ barn	0.91
Bohr magneton	$\mu_B = e\hbar/2m_e$	$5.788\ 381\ 8060(17) \times 10^{-11}$ MeV T^{-1}	0.30
nuclear magneton	$\mu_N = e\hbar/2m_p$	$3.152\ 451\ 258\ 44(96) \times 10^{-14}$ MeV T^{-1}	0.31
electron cyclotron freq./field	$\omega^e_{\text{cycl}}/B = e/m_e$	$1.758\ 820\ 010\ 76(53) \times 10^{11}$ rad s^{-1} T^{-1}	0.30
proton cyclotron freq./field	$\omega^p_{\text{cycl}}/B = e/m_p$	$9.578\ 833\ 1560(29) \times 10^{7}$ rad s^{-1} T^{-1}	0.31
gravitational constant‡	G_N	$6.674\ 30(15) \times 10^{-11}$ m^3 kg^{-1} s^{-2}	2.2×10^4
		$= 6.708\ 83(15) \times 10^{-39}\ \hbar c\ (\text{GeV}/c^2)^{-2}$	2.2×10^4
standard gravitational accel.	g_N	$9.806\ 65$ m s^{-2}	exact

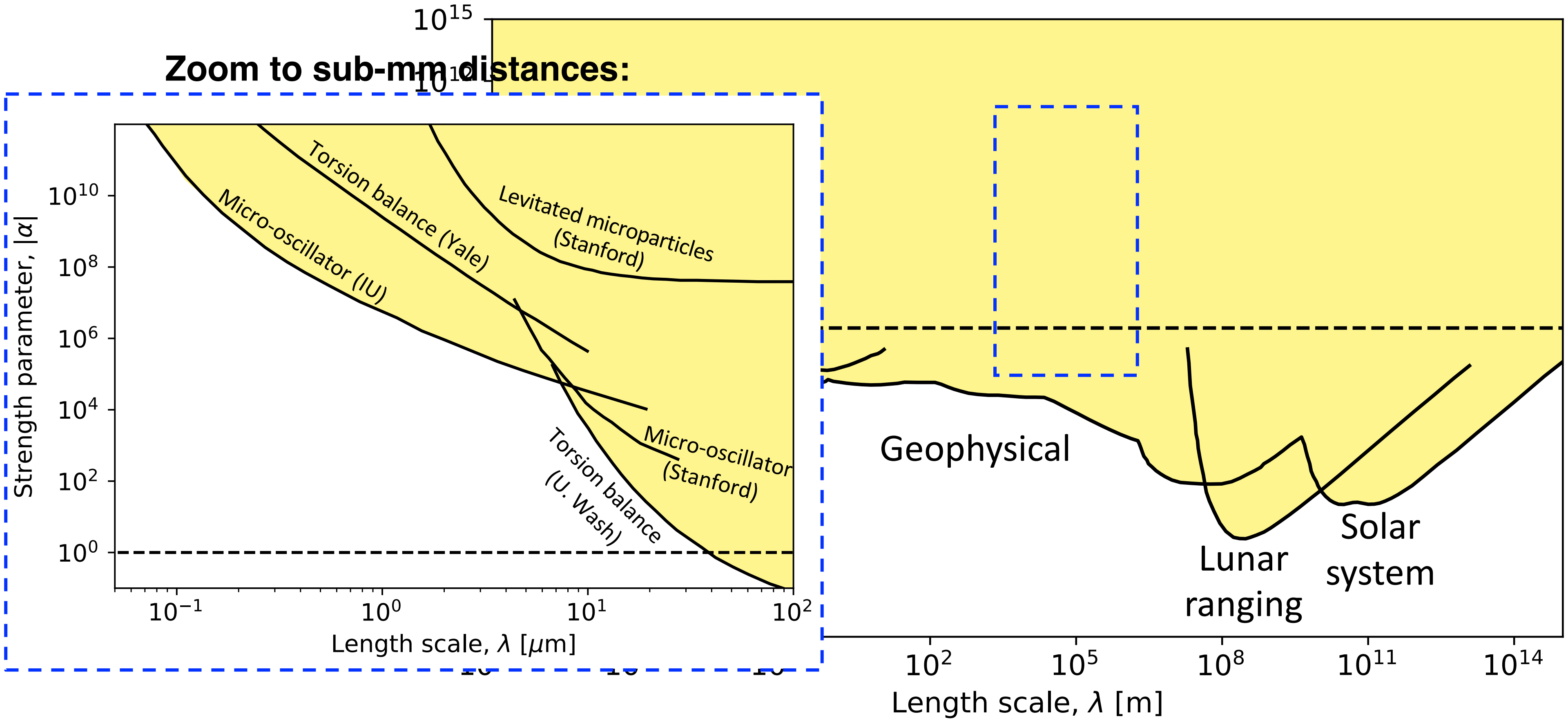
Short range forces

- More relevant for fundamental physics, the overall strength of gravity (or gravity-like interactions) can also be measured as a function of length scale:

$$V = \frac{G m_1 m_2}{r}\left(1 + \alpha e^{-r/\lambda}\right)$$

Fifth force coupling to mass?

Constraints on strength of gravity-like interactions vs length scale:



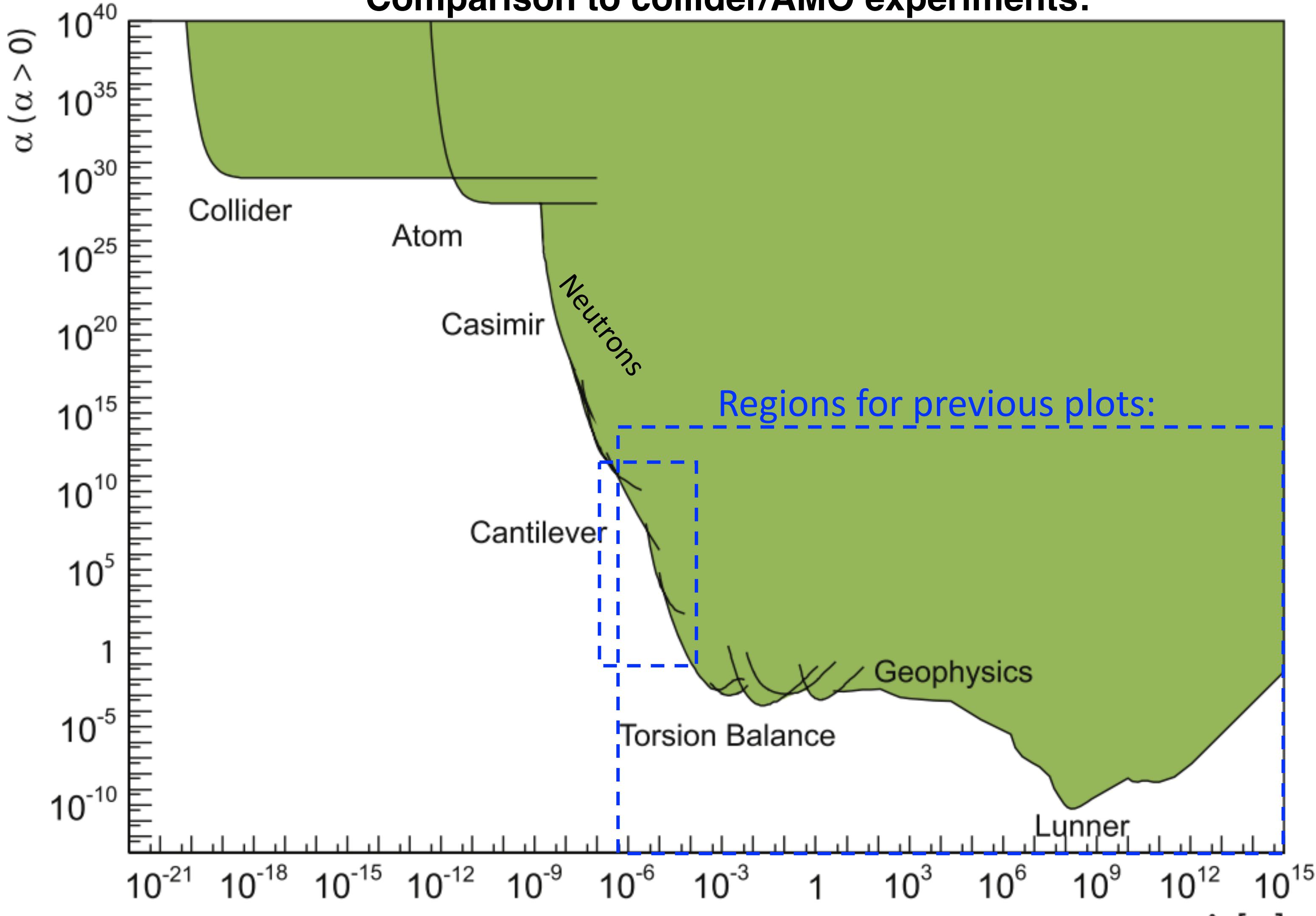
"Even-shorter" range forces

- Below μm distances, constraints on gravity-like forces rapidly fall off
- Best constraints arise from:

  1 nm – 1 μm:
  Casimir force measurements
  0.1 A – 1 nm:
  Atomic systems (i.e. atomic/Rydberg energy levels)
  <0.1 A:
  Colliders

- Gravity-strength interactions are much smaller than experimental sensitivity
- Beyond the required sensitivity, testing gravity at this scale limited by backgrounds (e.g. Casimir)

Comparison to collider/AMO experiments:



https://doi.org/10.1088/0264-9381/32/3/033001

Equivalence principle

- Beyond measuring the strength of gravity, we'd like to test if the coupling is identical for all materials
- Conceptually, the equivalence principle says that inertial and gravitational mass are the same:

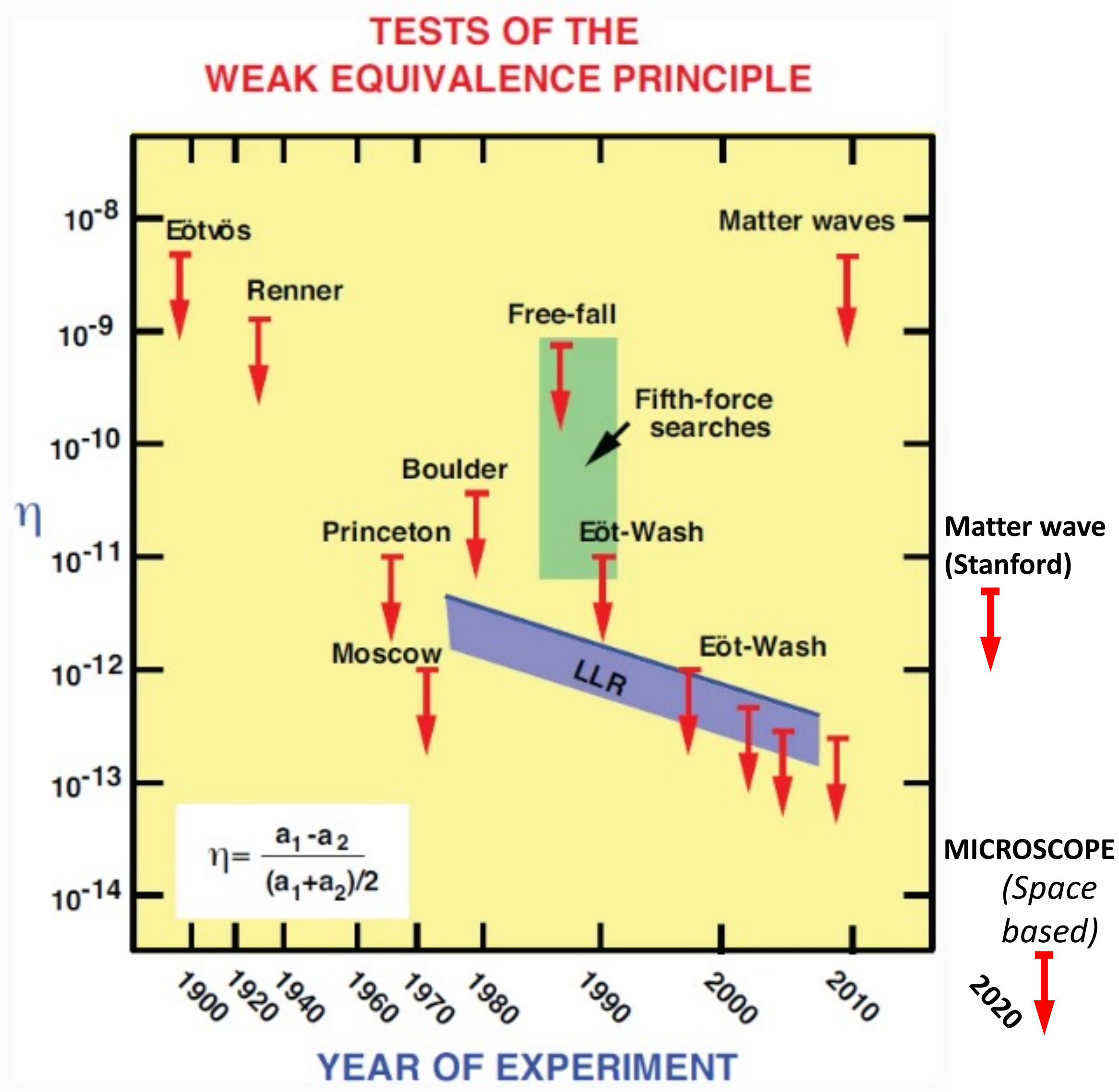
Are these identical?

$$m a = m |\vec{g}|$$

- This is equivalent to saying all materials feel the same gravitational force (not true for other interactions!)
- Famously Loránd Eötvös performed the first accurate test with a torsion balance (1885-1909)
- Constrain Eötvös parameter:

$$\eta_{1,2} = \frac{a_1 - a_2}{(a_1 + a_2)/2} = \frac{(m_g/m_i)_1 - (m_g/m_i)_2}{[(m_g/m_i)_1 + (m_g/m_i)_2\,]/2}$$

- Note in the lab we test the "weak" equivalence principle
    - "Strong" EP also includes gravitational self-binding energy



https://link.springer.com/article/10.12942/lrr-2014-4

Equivalence principle

- Equivalence principle violations may arise from a "5th force" rather than gravity itself:

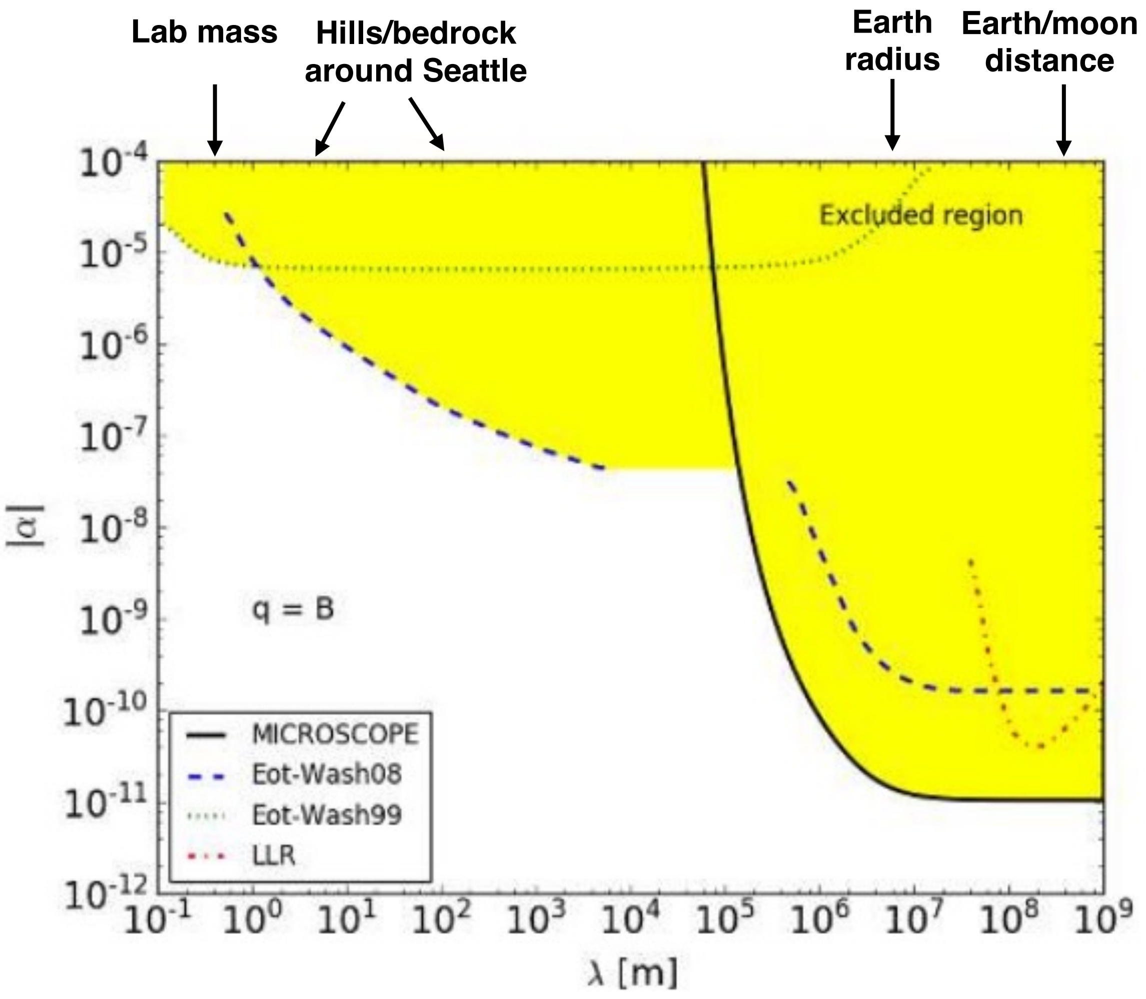
$$V = \frac{G m_1 m_2}{r}\left(1 + \alpha e^{-r/\lambda}\right)$$

- In this case, searches for EP violations should be performed as a function of length scale
- In general, the Eötvös parameter can be written in a length scale dependent form, e.g. for Pt/Ti:

$$\eta = \alpha\left[\left(\frac{q}{\mu}\right)_{\mathrm{Pt}} - \left(\frac{q}{\mu}\right)_{\mathrm{Ti}}\right]\left(\frac{q}{\mu}\right)_E\left(1+\frac{r}{\lambda}\right)e^{-r/\lambda}$$

- Generically the charges coupling to the force, *q*, can be B, B-L, ….

Length scale dependent tests of the weak EP:

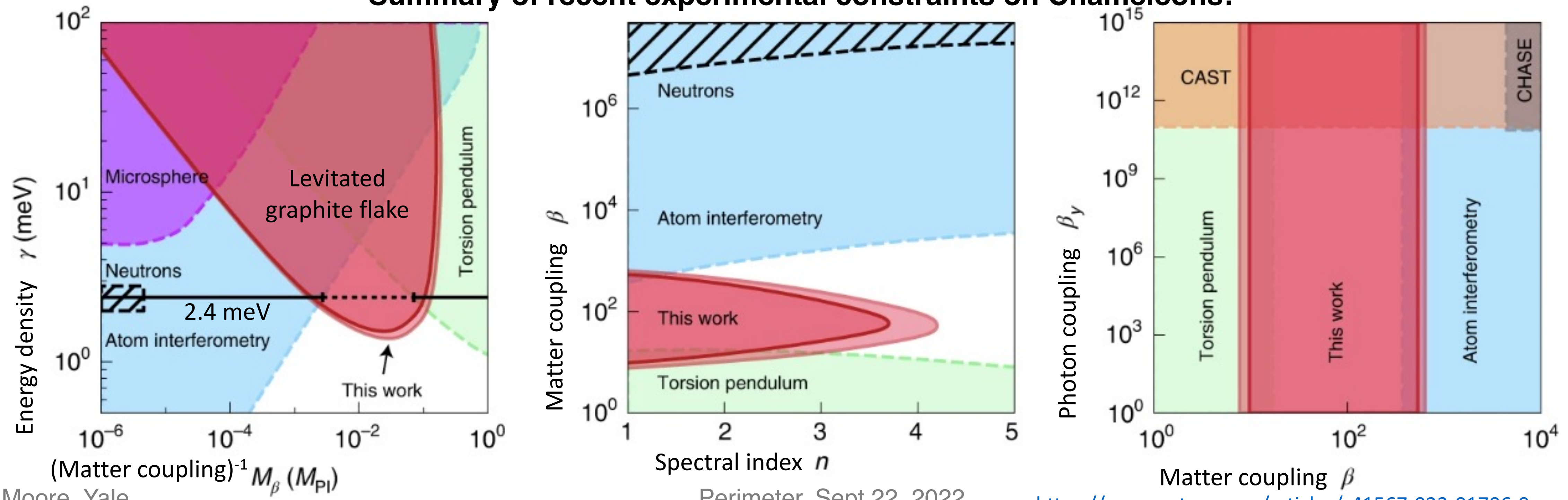


https://link.springer.com/article/10.12942/lrr-2014-4

"Screened" forces

- Finally, there has been recent interest in searches for 5th forces that do not follow the standard "Yukawa" potential
- These are typically motivated by dark energy models, in which Eot-Wash and other bounds are evaded by "screening mechanisms
- In the "Chameleon" model, the range of the force depends on the local mass density:
  - The standard Chameleon models have now been fully ruled out by laboratory tests (Atom interferometry, levitated systems)

Summary of recent experimental constraints on Chameleons:



https://www.nature.com/articles/s41567-022-01706-9

Summary

- Despite the fact that we've been studying gravity for longer than all the other fundamental forces, we have the weakest experimental constraints on its nature
  - This is due to its extreme weakness compared to the other forces (~10^{-40} the E&M force in a hydrogen atom)
- Torsion balances provided the first laboratory measurements of gravity, and experiments like Eot-Wash remain at the forefront of the field
- A number of new techniques are aiming to push these measurements to shorter distances or higher precision (next lecture!)