Life on the Lattice

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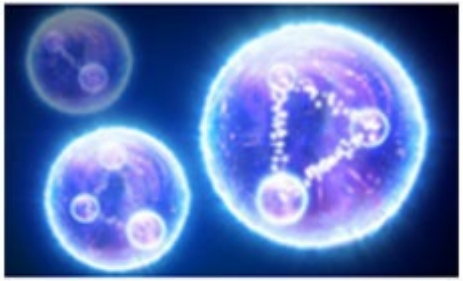
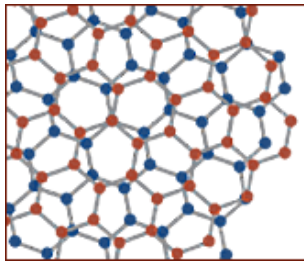


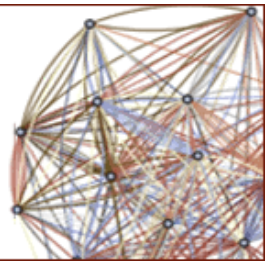
Simons Collaboration on

Global Categorical Symmetries



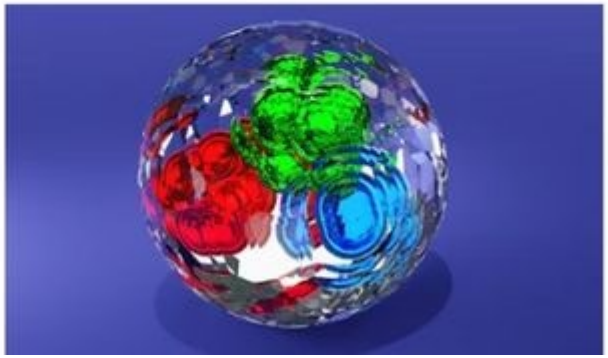
Simons Collaboration on

Ultra-Quantum Matter



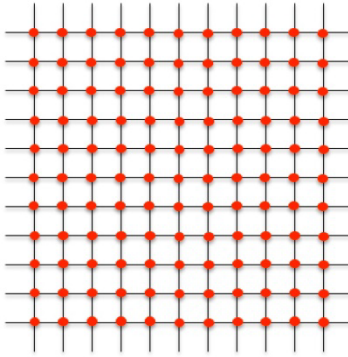
Simons Collaboration on

Confinement and QCD Strings



Lattice vs. continuum QFT

QFT is enormously successful. Yet, it is not mathematically rigorous.

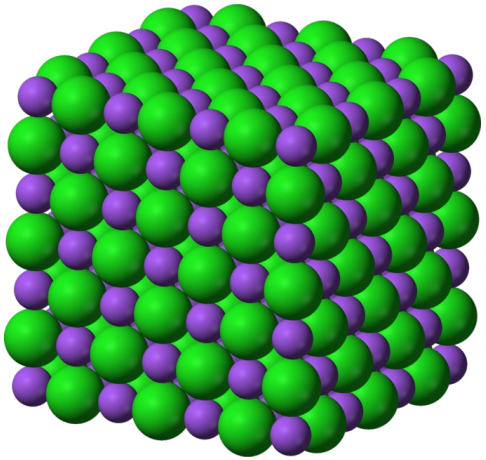


One approach is to regularize it by placing it on a lattice.

- Then, the problem is well defined.
- Continuum limit: introduce a lattice spacing a, take $a \to 0$ and the number of sites to infinity holding the physical lengths fixed – correlation functions at fixed positions $x \gg a \to 0$.
- Allows numerical calculations.

In condensed matter physics, the problem is defined on a (spatial or spacetime) lattice and the goal is to find the low-energy/long-distance limit.

- It is expected to be described by an effective continuum QFT.



From the continuum to the lattice – challenges

- Some continuum theories depend on the topology of field space, which relies on continuity. How is this captured by the lattice theory?

  This issue affects

  - Various terms in the action (e.g., θ-terms, Chern-Simons terms, Wess-Zumino terms, …)
  - Some global symmetries (e.g., winding symmetries, higher-form symmetries, non-invertible symmetries, …)
  - Anomalies
- Some QFTs (e.g., theories with self-dual forms or fermions) do not admit a suitable continuum Lorentz invariant action, and others (e.g., the 6d $(2,0)$ theory) do not even have a continuum Lagrangian at all. Not clear how to place them on the lattice.

From the lattice to the continuum – challenges

- What is the low-energy limit?
  - What are the possible phases and the transitions between them?
  - Which phases are connected?
    - Symmetries, anomalies
    - More criteria?
- Does the continuum limit exist? Does it depend on the microscopic details?
- Do all lattice models lead at long distances to a continuum QFT?

  This is particularly puzzling for various exotic models (e.g., fractons)
  - UV/IR mixing – long-distance phenomena depend on short-distance details. (Reminiscent of quantum gravity and some string theory constructions.)

No conclusions

Thank you