Life on the lattice, Part II

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The mysterious effectiveness of QFT in condensed matter

- Critical points of classical lattice models are often described using Euclidean Conformal Field Theory (CFT)
- Gapless phases are often described by a local QFT (but not necessarily Lorentz-invariant)
- Gapped phases are often described by TQFT (hence one often conflates "gapped" and "topological")

Why should this be the case?

What are the limitations of QFT?

Are there QFTs which do not arise from lattice models?

Examples

- IQHE: response of a disordered 2D electron gas in a strong magnetic field is described by a $U(1)$ Chern-Simons action:

$$S_{3d} = \frac{k}{4\pi} \int_M A \wedge dA$$

- Symmetry-Protected Topological (SPT) phases in d dimensions are described by actions which may involve characteristic classes of $(d+1)$-manifolds: for example, the non-trivial phase of 1d spin chains with time-reversal symmetry corresponds to

$$S_{2d} = i\pi \int_N w_1 \cup w_1$$

Counter-examples

- Non-local microscopic interactions may lead to critical behavior which is not described by a local QFT (e.g. critial point of the classical Ising spin chain with $1/r^\alpha$ interactions, $1 < \alpha \leq 2$).
- Fractons: gapped Hamiltonians which have localized finite-energy excitations which are not "mobile" (translations do not preserve "super-selection sector" of excitations).
- Chiral TQFTs (such as quantum Chern-Simons theory): these are hard to "put on a lattice"

Conceptual issue: how does one check that a given lattice model is a lattice version of a TQFT?

Opportunities

Lattice models can be studied in mathematically rigorous ways. Can help to shed some light on long-standing QFT issues and perhaps even provide some ideas for a rigorous definition of QFT.

- Anomalies and the bulk-boundary correspondence
  For 1+1d theories, 't Hooft anomaly manifests itself on the operator level (charge density Lie algebra is centrally extended). In 3+1d theories, no operator formulation of 't Hooft anomalies is known. Can one understand this issue by thinking about a QFT with an 't Hooft anomaly as living on the edge of a gapped lattice system?
- Higher symmetries and higher (categorical) structures in QFT
  Symmetry in QFT is not described by a group (or a Lie algebra), but a "(d+1)-group". Can one identify this higher group as a symmetry of the bulk lattice model?