Black hole cohomologies in $\mathcal{N} = 4$ Yang-Mills

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Talk based on collaborations with

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“The shape of non-graviton operators for SU(2)” arXiv:2209.12696.
“Towards quantum black hole microstates” arXiv.2304.10155.

See also:

- Chi-Ming Chang, Ying-Hsuan Lin,
  “Words to describe a black hole” arXiv:2209.06728.

- Budzik, Gaiotto, Kulp, Williams, Wu, Yu,
  “Semi-chiral operators in 4d N=1 gauge theories” arXiv:2306.01039.
- Chang, Feng, Lin, Tao,
  “Decoding stringy near-supersymmetric black holes” arXiv:2306.04673.
- Budzik, Murali, Vieira,
  “Following black hole states” arXiv:2306.04693.

The problem & motivations

The problem: 1/16-BPS states in 4d maximal SYM w/ SU(N) gauge group

- Among $16Q + 16S$ supercharges, pick a pair Q & $S = Q^\dagger$ that the BPS states preserve.
- BPS states saturate the bound: $\{Q, Q^\dagger\} \sim E - (R_1 + R_2 + R_3 + J_1 + J_2) \geq 0$
- Nilpotency $Q^2 = 0$: BPS states ~ harmonic forms $\xleftrightarrow{\text{1 to 1}}$ Q-cohomology classes.
- We shall find & discuss the representatives of new cohomology classes.

Motivations:

- Special case of $1/4$-BPS states in $N = 1$ QFT: Information on SUSY dynamics
  [Budzik, Gaiotto, Kulp, Williams, Wu, Yu] (2023)
- Our main motivation: BPS black hole microstates in $AdS_5 \times S^5$
  → Requires studying strong coupling & large N.

I will explain recent (perhaps modest) progress in this program.

- Cohomologies at weak-coupling (1-loop ~ $O(g_{YM}^2)$)
- Want to eventually study $SU(N \gg 1)$. ↔ Today, I will report SU(2) & a bit of SU(3).
- Qualitative features & rough comparison w/ “gravity dual” (← mostly omitted today)

Historical remarks

2004~ : [Gutowski, Reall] … Constructions of BPS black holes in $AdS_5 \times S^5$

2005~ : [Romelsberger] [Kinney, Maldacena, Minwalla, Raju] …

Efficient way to study BPS spectrum → the "superconformal index"

- At that time, it was unclear how to see black holes from this index.
- Direct studies w/ weak-coupling cohomology: No new solutions found beyond BPS gravitons.
  [Berkooz, Reichmann, Simon] (2006) [Grant, Grassi, SK, Minwalla] (2008) [Chang, Yin] (2013) …

2018 ~ : [Cabo Bizet, Cassani, Martelli, Murthy] [Choi, J. Kim, SK, Nahmgoong] [Benini, Milan] …

Realized how to see black holes from the index. (I.e. computed their entropy.)

- This index is coupling-independent → Counts weak-coupling cohomologies.
- Since it counted black holes, we became confident that new cohomologies should exist.

2022 ~ : [Chang, Lin] [Choi, SK, E. Lee, Park] …

With these recent realizations & promises, revisited the cohomology problem.

- Especially because the cohomologies carry more information than the index.

Weak-coupling setup

SU(N) maximal SYM on R^4: all fields in adjoint rep. (written in N=1 language)

3 chiral multiplets: ϕ_m , $\bar{\phi}^m$ and $\psi_{m\alpha}$, $\bar{\psi}^m_{\dot{\alpha}}$ $(m = 1,2,3)$

vector multiplet: $A_\mu \sim A_{\alpha\dot{\beta}}$ and λ_α , $\bar{\lambda}_{\dot{\alpha}}$ $(\mu = 1, \cdots, 4)$ $(\alpha = \pm, \dot{\alpha} = \dot{\pm})$

Gauge-invariant local BPS operators: (at $x^\mu = 0$ on R^4)

- Free limit ($g_{YM} \to 0$): Any gauge-invariants of the invariant fields under Q & $Q^\dagger$:

$\boxed{\bar{\phi}^m,\ \psi_{m+},\ \bar{\lambda}_{\dot{\alpha}},\ f_{++} \equiv F_{1+i2\,,\,3+i4}\ \text{\& derivatives } \partial_{1+i2} \equiv \partial_1 - i\partial_2\ ,\ \partial_{3+i4} \equiv \partial_3 - i\partial_4 \text{ acting on them}}$

- Not all of them are BPS at $g_{YM} \neq 0$: At small $g_{YM} \ll 1$, classical SUSY on them reads

$\boxed{Q\,\bar{\phi}^m = 0\ ,\ Q\psi_{m+} \sim g_{YM}\epsilon_{mnp}[\bar{\phi}^n, \bar{\phi}^p]\ ,\ Qf_{++} \sim g_{YM}[\psi_{m+}, \bar{\phi}^m]\ ,\ Q\,\bar{\lambda}_{\dot{\alpha}} = 0\ ,\ \ [Q, D_{+\dot{\alpha}}] \sim g_{YM}[\,\bar{\lambda}_{\dot{\alpha}}\,,\ \ \}}$

- Classical Q & $Q^\dagger$ at ½-loop → Anomalous dimension $QQ^\dagger + Q^\dagger Q \sim E - E_{BPS}$ at 1-loop, $O(g_{YM}^2)$.

Goal: Find free BPS operators which remain BPS at 1-loop level.

- Remain BPS at strong-coupling? A non-renormalization theorem pursued. [Chang, Lin] (2022)
- But their index captures BH's → So empirically, many of them should remain BPS.
- So expecting "certain" non-renormalization, we study 1-loop cohomologies.

The strategy

The problem at finite $N = 2,3,\cdots$:

- Construct all cohomologies at given charges, and mod out those from BPS multi-gravitons

  → [Chang, Lin] (2022) did it & found the first non-graviton cohomology for SU(2).

What are the "BPS gravitons at finite N" …?

- Chiral primaries $\mathrm{tr}[\bar{\phi}^{(m_1} \cdots \bar{\phi}^{m_n)}]$, descendants (1-particle) & their products (multi-particle)
- Finite N subtlety: # of independent operators reduces due to trace relations

  → QFT dual of "stringy exclusion principle" due to giant gravitons

  [Maldacena, Strominger] (1998)
  [Jevicki, Ramgoolam] (1999)
  [Ho, Ramgoolom, Tatar] (1999)
  [Susskind, McGreevy, Toumbas] (2000)
  [Grisaru, Myers, Tafjord] (2000)
  [Hashimoto, Hirano, Itzhaki] (2000)
- So they reflect all the expected BPS graviton physics, including the finite N effects.

Remainders have chances to describe black holes. (→ Call "black hole operators")

- Conservatively, results at low $N = 2,3,\cdots$ are just intermediate steps towards large N.
- Progressively, lessons on black holes in wildly quantum ($G_N \sim 1/N^2$) gravity?

A streamlined strategy

To quickly diagnose which charge sectors host new operators, we use the index:

- Grade operators with charges appearing in the index, say with

$$j \equiv 6(R+J) = 2(R_1+R_2+R_3) + 3(J_1+J_2) \geq 0.$$

- Compute the full index & that over gravitons → Subtract. [Choi, SK, E. Lee, S. Lee, Park] (2023)

SU(2) → Study the index $Z(t) = Tr[(-1)^F t^j]$

- In full generality, only have series expansions till certain order:

$$\begin{aligned} Z(t) &= 1+6t^4-6t^5-7t^6+18t^7+6t^8-36t^9+6t^{10}+84t^{11}-80t^{12}-132t^{13}+309t^{14}-18t^{15}-567t^{16} \\ &\quad +516t^{17}+613t^{18}-1392t^{19}-180t^{20}+2884t^{21}-1926t^{22}-4242t^{23}+7890t^{24}+792t^{25}-15876t^{26} \\ &\quad +13804t^{27}+15177t^{28}-37536t^{29}+7049t^{30}+57522t^{31}-58704t^{32}+\cdots \\ Z_{\text{grav}}(t) &= 1+6t^4-6t^5-7t^6+18t^7+6t^8-36t^9+6t^{10}+84t^{11}-80t^{12}-132t^{13}+309t^{14}-18t^{15}-567t^{16} \\ &\quad +516t^{17}+613t^{18}-1392t^{19}-180t^{20}+2884t^{21}-1926t^{22}-4242t^{23}+7891t^{24}+786t^{25}-15864t^{26} \\ &\quad +13804t^{27}+15138t^{28}-37476t^{29}+7048t^{30}+57414t^{31}-58566t^{32}+\cdots \end{aligned}$$

$$\boxed{Z - Z_{\text{grav}} = -t^{24}+6t^{25}-12t^{26}+0t^{27}+39t^{28}-60t^{29}+t^{30}+108t^{31}-138t^{32}+\cdots}$$

- Computed exactly in "BMN truncation":

$$Q\bar{\phi}^m = 0\ ,\ \ Q\psi_{m+} \sim \epsilon_{mnp}[\bar{\phi}^n, \bar{\phi}^p]\ ,\ \ Qf_{++} \sim [\psi_{m+}, \bar{\phi}^m]$$

[Berenstein, Maldacena, Nastase] (2002)

[Nakwoo Kim, Klose, Plefka] (2003)

$$[Z - Z_{\text{grav}}]_{\text{BMN}} = -\frac{t^{24}}{1-t^{12}} \cdot \frac{1}{(1-t^8)^3} \cdot (1-t^2)^3$$

Constructions

An ∞-tower of non-graviton cohomologies, whose representatives are: $n = 0,1,2,\cdots$

$$
\begin{aligned}
O_n \;=\; & (f\cdot f)^n \epsilon^{c_1c_2c_3}(\phi^a\cdot\psi_{c_1})(\phi^b\cdot\psi_{c_2})(\psi_a\cdot\psi_b\times\psi_{c_3}) \\
& +n(f\cdot f)^{n-1}\epsilon^{b_1b_2b_3}\epsilon^{c_1c_2c_3}(f\cdot\psi_{b_1})(\phi^a\cdot\psi_{c_1})(\psi_{b_2}\cdot\psi_{c_2})(\psi_a\cdot\psi_{b_3}\times\psi_{c_3}) \\
& -\left(\frac{n}{72}+\frac{n(n-1)}{108}\right)(f\cdot f)^{n-1}\epsilon^{a_1a_2a_3}\epsilon^{b_1b_2b_3}\epsilon^{c_1c_2c_3}(\psi_{a_1}\cdot\psi_{b_1}\times\psi_{c_1})(\psi_{a_2}\cdot\psi_{b_2}\times\psi_{c_2})(\psi_{a_3}\cdot\psi_{b_3}\times\psi_{c_3})
\end{aligned}
$$

- In particular, the "threshold" non-graviton operator at t^{24} order:

$$O_0 = \epsilon^{p_1p_2p_3}(\phi^m\cdot\psi_{p_1})(\phi^n\cdot\psi_{p_2})(\psi_m\cdot\psi_n\times\psi_{p_3})$$

(Used 3d vector notation for SU(2) adjoints: $A\cdot B \sim \mathrm{tr}(AB)$ and $A\times B\sim[A,B]$)

This tower fully explains the BMN index:

$$[Z-Z_{\text{grav}}]_{\text{BMN}} = -\frac{t^{24}}{1-t^{12}}\cdot\frac{1}{(1-t^8)^3}\cdot(1-t^2)^3$$

"core black hole" primary operators O_n

descendants within BMN

Limited dressings by gravitons $tr(2\bar{\phi}^m f+\epsilon^{mnp}\psi_n\psi_p)$
(only 3 out of 17 gravitons in BMN sector)

- These operators don't really look like black holes in most senses. (E.g. small entropy)
- But they exhibit subtle properties, qualitatively reminiscent of black holes. (→ next slide)

A no-hair theorem?

To appreciate the last point, see the BMN results:

- Q satisfies Leibniz rule → The product (BH) x (graviton) is another cohomology.
- There are 17 different species of graviton particles in the BMN sector.
- But "black hole operators" O_n abhor dressings by all but 3 gravitons: $tr(2\bar{\phi}^m f + \epsilon^{mnp}\psi_n\psi_p)$.

$$[Z - Z_{\rm grav}]_{\rm BMN} = \boxed{-\frac{t^{24}}{1-t^{12}}} \cdot \boxed{\frac{1}{(1-t^8)^3}} \cdot (1-t^2)^3$$

This feature continues in the general SU(2) index:

- Eliminating $PSU(1,2|3) \subset PSU(2,2|4)$ superconformal descendants of O_0, the remainder is:

$$\chi_0(t) = -t^{24} \boxed{+6t^{25} - 12t^{26} + 0t^{27} + 39t^{28} - 60t^{29} + t^{30} + 108t^{31}} - 135t^{32} + \cdots$$

$$Z - Z_{\rm grav} - \chi_0(t) = -3t^{32} + \cdots$$

- The "boring" range $25 \leq j \leq 31$ → Many product cohomologies absent in the index.
- Simplest possibility: All Q-exact, i.e. absent ← Checked explicitly for many (next slide).

Only a partial no-hair theorem:

- Conformal primaries of gravitons: 29 of 32 don't dress O_0 (at least invisible in the index).
- Conformal descendants…? (more in our paper → next page)

Illustration: Q-exactness at low levels

t^{28}:

$$
\begin{aligned}
O_0(\bar\phi^{(m}\cdot\bar\phi^{n)}) = -\frac{1}{14}Q[&20\epsilon^{rs(m}(\bar\phi^{n)}\cdot\psi_{p+})(\bar\phi^{p}\cdot\psi_{r+})(\bar\phi^{q}\cdot\psi_{q+})(f_{++}\cdot\psi_{s+})\\
&-20\epsilon^{prs}(\bar\phi^{(m}\cdot\psi_{p+})(\bar\phi^{n)}\cdot\psi_{r+})(\bar\phi^{q}\cdot\psi_{q+})(f_{++}\cdot\psi_{s+})\\
&+30\epsilon^{prs}(\bar\phi^{(m}\cdot\psi_{p+})(\bar\phi^{n)}\cdot\psi_{r+})(\bar\phi^{q}\cdot\psi_{s+})(f_{++}\cdot\psi_{q+})\\
&-7\epsilon^{a_1a_2p}\epsilon^{b_1b_2(m}(\bar\phi^{n)}\cdot\psi_{p+})(\bar\phi^{q}\cdot\psi_{q+})(\psi_{a_1+}\cdot\psi_{a_2+})(\psi_{b_1+}\cdot\psi_{b_2+})\\
&+18\epsilon^{a_1a_2p}\epsilon^{b_1b_2(m}(\bar\phi^{n)}\cdot\psi_{q+})(\bar\phi^{q}\cdot\psi_{p+})(\psi_{a_1+}\cdot\psi_{a_2+})(\psi_{b_1+}\cdot\psi_{b_2+})]
\end{aligned}
$$

t^{29}:

$$
\begin{aligned}
O_0(\bar\phi^{m}\cdot\bar\lambda_{\dot\alpha}) = \frac{1}{8}Q[&40\epsilon^{mnp}(f_{++}\cdot\psi_{q+})(\bar\lambda_{\dot\alpha}\cdot\psi_{r+})(\bar\phi^{q}\cdot\psi_{n+})(\bar\phi^{r}\cdot\psi_{p+})\\
&-4\epsilon^{ma_1a_2}\epsilon^{nb_1b_2}(\bar\lambda_{\dot\alpha}\cdot\psi_{n+})(\bar\phi^{p}\cdot\psi_{p+})(\psi_{a_1+}\cdot\psi_{a_2+})(\psi_{b_1+}\cdot\psi_{b_2+})\\
&+6\epsilon^{ma_1a_2}\epsilon^{nb_1b_2}(\bar\lambda_{\dot\alpha}\cdot\psi_{p+})(\bar\phi^{p}\cdot\psi_{n+})(\psi_{a_1+}\cdot\psi_{a_2+})(\psi_{b_1+}\cdot\psi_{b_2+})\\
&+\epsilon^{na_1a_2}\epsilon^{pb_1b_2}(\bar\lambda_{\dot\alpha}\cdot\psi_{n+})(\bar\phi^{m}\cdot\psi_{p+})(\psi_{a_1+}\cdot\psi_{a_2+})(\psi_{b_1+}\cdot\psi_{b_2+})]
\end{aligned}
$$

t^{30}:

$$
\begin{aligned}
&O_0\left(\bar\phi^{m}\cdot\psi_{n+}-\tfrac{1}{3}\delta^{m}_{n}\bar\phi^{p}\cdot\psi_{p+}\right)\\
&=\frac{1}{4}Q\left[\epsilon_{npq}\epsilon^{ra_1a_2}\epsilon^{qb_1b_2}\epsilon^{mc_1c_2}(\bar\phi^{p}\cdot\psi_{r+})(\psi_{a_1+}\cdot\psi_{a_2+})(\psi_{b_1+}\cdot\psi_{b_2+})(\psi_{c_1+}\cdot\psi_{c_2+})\right]
\end{aligned}
$$

Concluding remarks

"Construction" (in a limited sense) of BPS black hole microstates

- Found new "black hole cohomologies" for SU(2)
- What I presented are not actual BPS states, even at 1-loop. Just representatives.

  [Chang, Feng, Lin, Tao] (2023) [Budzik, Murali, Vieira] (2023)
- SU(3) in BMN sector [Jae Hyeok Choi, Jehyun Lee, Siyul Lee] (work in progress)

$$Z - Z_{grav} = \boxed{-t^{24}} + 3t^{26} - 3t^{28} - 10t^{30} + 15t^{32} + 24t^{34} - 36t^{36} - 45t^{38} + 39t^{40} + 124t^{42} + O(t^{44})$$

Same threshold level as SU(2)...!

- Higher N? Higher charges? Analytic structures? Insights from emergent structures of holomorphically twisted QFT? … [Budzik, Gaiotto, Kulp, Williams, Wu, Yu] (2023)

Better picture on hairy (BPS) black holes in $AdS_5 \times S^5$?

[Bhattacharyya, Minwalla, Papadodimas] (2010) [Markeviciute, Santos] [Markeviciute] (2018)

[Niehoff, Santos, Way] (2015) [Chesler] (2021) [SK, Kundu, E. Lee, J. Lee, Minwalla, Patel] (2023) …

- Studied BPS perturbations of BPS black holes, dual to $(\partial_{++})^{m_1}(\partial_{+-})^{m_2}\mathrm{tr}(X^2+Y^2+Z^2)$

  Similar "partial no-hair" behavior as SU(2) cohomologies. [Choi, SK, E. Lee, S. Lee, Park] (2023)

$$\begin{cases} \text{Perturbative dressing forbidden:} & m_1 + m_2 < 4q/\ell^2 \\ \text{Allowed (large conformal descendants):} & m_1 + m_2 \geq 4q/\ell^2 \end{cases}$$

q: "size" parameter of BH
ℓ: AdS radius