Hilbert space and holography of information in de Sitter quantum gravity

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Strings 2023
Perimeter Institute
26 July 2023

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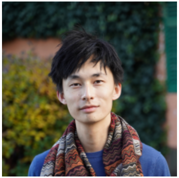
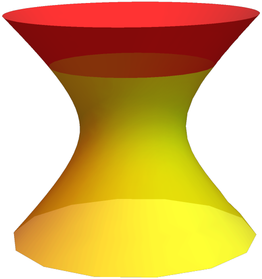


Victor Godet

- `2303.16316` and `2303.16315`, Joydeep Chakravarty, Tuneer Chakraborty, Victor Godet, Priyadarshi Paul, S.R.

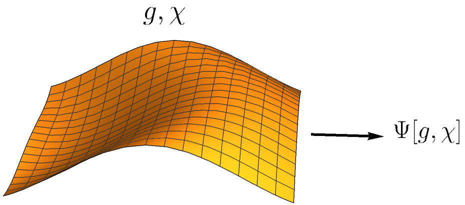
Questions



Focus on asymptotically dS spacetime from a global perspective.

- What is the Hilbert space for gravity in such a spacetime?
- Gravity localizes information unusually. How does holography of information work in such a spacetime?

Wavefunctionals



States can be represented as wavefunctionals on the late-time slice.

$\Psi[g, \chi]$ assigns an amplitude to a configuration of

metric on a spacelike slice $\quad g$

and

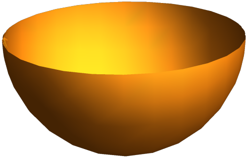
matter fields $\quad \chi$

Vacuum wavefunctional

- We understand the Euclidean vacuum state well.

$$|0\rangle \leftrightarrow \Psi_0[g, \chi]$$

- Computed using the Hartle-Hawking proposal



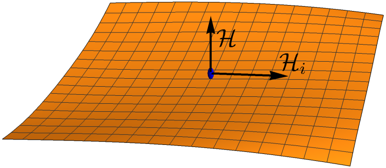
[Hartle,Hawking, 1983]

- Also computed via analytic continuation from AdS

$$Z_{\text{CFT}}[g, \chi] \to \Psi_0[g, \chi]$$

[Maldacena, 2001]

Constraints of gravity



Wavefunctionals in quantum gravity obey

$$\mathcal{H}\Psi[g, \phi] = 0; \qquad \mathcal{H}_i\Psi[g, \phi] = 0.$$

Procedure: Solve for the Hilbert space by finding a complete basis of solutions to the WDW equation.

WDW equation

Explicitly,

$$\mathcal{H} = 2\kappa^2 g^{-1}\left(g_{ik}g_{jl}\pi^{kl}\pi^{ij} - \frac{1}{d-1}(g_{ij}\pi^{ij})^2\right) - \frac{1}{2\kappa^2}(R - 2\Lambda)$$

$$+ \mathcal{H}_{\text{matter}} + \mathcal{H}_{\text{int}},$$

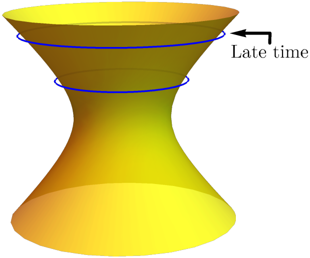
$$\mathcal{H}_i = -2g_{ij}D_k\frac{\pi^{jk}}{\sqrt{g}} + \mathcal{H}_i^{\text{matter}},$$

Simplifying the WDW equation

- In the regime

$$\Lambda \gg R; \qquad \Lambda \gg V_{\text{matter}}$$

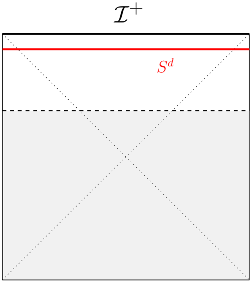
the WDW equation turns out to be tractable.

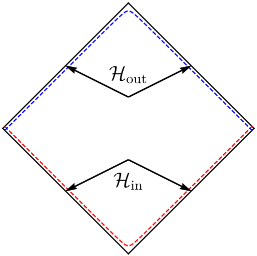


The limit $\Lambda \gg R$ focuses us on the late-time slice.

Late-time limit

- Solving WDW at "large volume" gives us "late time" behaviour of the state.
- Sufficient to understand Hilbert space. (cf. asymptotic quantization).
- Insufficient for bulk dynamics/"finite-time physics".





Solution

At large volume all solutions of the WDW equation take the form

$$\Psi \longrightarrow e^{iS[g,\chi]} Z[g,\chi]$$

see AdS solutions by Freidel (2008),Regado, Khan, Wall (2022)

1. S is a divergent universal phase factor.
2. $Z[g,\chi]$ is diff invariant and almost Weyl invariant

$$\Omega \frac{\delta Z[g,\chi]}{\delta \Omega(x)} = \mathcal{A}_d[g] Z[g,\chi].$$

   $\mathcal{A}_d$ is an imaginary local function of g in even d for dS_{d+1}.
3. $$|Z[g,\chi]|^2$$

   is Weyl invariant.

Phase factor

The phase factor S contains terms familiar from holographic renormalization.

$$S = -\frac{(d-1)}{\kappa^2}\int \sqrt{g} d^d x + \frac{1}{2\kappa^2(d-2)}\int \sqrt{g} R d^d x + \dots$$

[Papadimitriou, Skenderis, 2004]

It comprises integrals of local densities.

It doesn't depend on details of state.

Cancels out in $|\Psi[g,\chi]|^2$.

Expansion of $Z[g,\chi]$

After Weyl transformation to frame

$$g_{ij} = \delta_{ij} + \kappa h_{ij},$$

Expand

$$Z[g,\chi] = \exp[\sum_{n,m} \kappa^n \mathcal{G}_{n,m}]$$

with

$$\mathcal{G}_{n,m} = \int d\vec{y}\, d\vec{z}\, G^{\vec{i}\vec{j}}_{n,m}(\vec{y},\vec{z}) h_{i_1 j_1}(z_1) \dots h_{i_n j_n}(z_n) \chi(y_1) \dots \chi(y_m),$$

Coefficient fns obey same Ward identities as CFT correlators.

$$G^{\vec{i}\vec{j}}_{n,m}(\vec{y},\vec{z}) \sim \langle T^{i_1 j_1}(y_1) \dots T^{i_n j_n}(y_n) \phi(z_1) \dots \phi(z_m) \rangle^{\text{connected}}_{\text{CFT}},$$

"CFT" is not unitary; not even necessarily local.

Hartle-Hawking state and other states

$$\Psi_0 = e^{iS} \exp[\sum_{n,m} \kappa^n \mathcal{G}_{n,m}]$$

[Pimentel, 2013]

[Hartle, Hawking, Hertog, 2008]

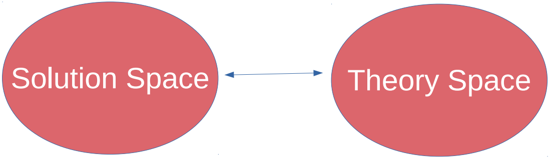
Not just the Hartle-Hawking state but all states have this form.

Interactions do not constrain precise form of $\mathcal{G}_{n,m}$ beyond conformal invariance of coefficient fns.

Solution space as theory space

List of correlators$\{G^{\vec{i}\vec{j}}_{n,m}(\vec{y},\vec{z})\} \longrightarrow$ WDW solution

But list of correlators can be thought of as defining a “theory”.



Caution: Additional constraints on allowed states come from normalizability.

Small fluctuations basis for states

Starting with $\mathcal{G}_{n,m}$ for H.H. state,

$$\mathcal{G}^{\lambda}_{n,m} = (1-\lambda)\mathcal{G}_{n,m} + \lambda\widetilde{\mathcal{G}}_{n,m}$$

Then

$$\frac{\partial \Psi_\lambda[g,\chi]}{\partial\lambda}\Big|_{\lambda=0} = \sum_{n,m} \kappa^n \delta\mathcal{G}_{n,m}\Psi_0[g,\chi]$$

$$= \sum_{n,m} \kappa^n \int d\vec{x}\, G^{\vec{i}\vec{j}}_{n,m}(\vec{y},\vec{z}) h_{i_1 j_1}(z_1)\dots h_{i_n j_n}(z_n)\chi(y_1)\dots\chi(y_m)\Psi_0[g,\chi]$$

Summary: solution space

$$|\Psi\rangle = \sum_{n,m} \kappa^n \int d\vec{y}\, d\vec{z}\, \delta G^{\vec{i}\vec{j}}_{n,m}(\vec{y},\vec{z})\, h_{i_1 j_1}(z_1) \ldots h_{i_n j_n}(z_n)\, \chi(y_1) \ldots \chi(y_m) |0\rangle$$

Matter fluctuation

Smearing functions satisfy CFT Ward identities

Metric fluctuation

Euclidean vacuum

Higuchi states

$$\Psi = \sum_{n,m} \kappa^n \int d\vec{x}\, \delta G^{\vec{i}\vec{j}}_{n,m}(\vec{y},\vec{z}) h_{i_1 j_1}(z_1) \ldots h_{i_n j_n}(z_n) \chi(y_1) \ldots \chi(y_m) \Psi_0$$

- The Ward identities tell us

$$\delta\mathcal{G}_{n,m} \neq 0 \implies \delta\mathcal{G}_{n+1,m} \neq 0.$$

- When $\kappa \to 0$, Ward identities do not relate $\delta\mathcal{G}_{n,m}$ to $\delta\mathcal{G}_{n+1,m}$.

$$|\Psi_{\text{ng}}\rangle = \int d\vec{y} f(y_1, \ldots y_n) \chi(y_1) \ldots \chi(y_n) |0\rangle$$

where f has the symmetries of a conformal correlator.

This can be shown to match a previous construction of the state space by Higuchi when $\kappa \to 0$.

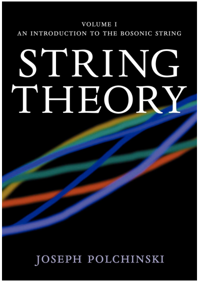
[Higuchi, 1991]
[Marolf, Morrison, 2008]
[Anninos, Denef, Monten, Sun, 2017]
[Chandrasekaran,Longo,Penington,Witten, 2022]

Proposal for norm

We propose

$$(\Psi, \Psi) = \frac{1}{\text{vol(diff}\times\text{Weyl)}} \int DgD\chi \sum_{n,m,n',m'} \kappa^{n+n'} \delta\mathcal{G}^*_{n,m} \delta\mathcal{G}_{n',m'} |Z_0[g,\chi]|^2$$

Proposal is not unique. But natural and simple.



Fixing gauge

$$\text{Fix gauge}: \qquad \sum_i \partial_i g_{ij} = 0; \qquad \delta^{ij} g_{ij} = d$$

Gauge choice leaves behind residual global transformations.

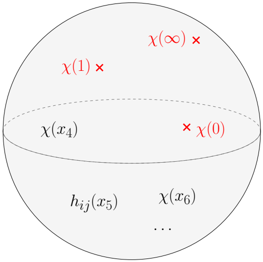
$$\begin{aligned} \text{translations}: &\quad \xi^i = \alpha^i; \\ \text{rotations}: &\quad \xi^i = M^{ij} x^j \\ \text{dilatations}: &\quad \xi^i = \lambda x^i \\ \text{SCTs}: &\quad \xi^i = (2(\beta \cdot x) x^i - x^2 \beta^i) + v^i_j \beta^j \end{aligned}$$

SCTs are corrected by a metric-dependent term for $d > 2$.

[Hinterbichler, Hui, Khoury, 2013]

[Ghosh, Kundu, S.R., Trivedi, 2014]

Fixing residual gauge freedom



Fix residual transformations by fixing positions of "vertex operators" in $\delta\mathcal{G}_{n,m}$.

$$x_1 = 0, \qquad x_2 = 1 \qquad x_3 = \infty$$

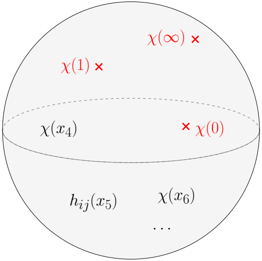
Finally

$$(\Psi, \Psi) = \sum_{n,m,n',m'} \kappa^{n+n'} \left\langle\!\left\langle \overline{\delta\mathcal{G}^*_{n,m}\,\delta\mathcal{G}_{n',m'}} \right\rangle\!\right\rangle$$

$$= \sum_{n,m,n',m'} \kappa^{n+n'} \int Dg D\chi\, \delta(\mathrm{g.f})\Delta'_{\mathrm{FP}}|Z_0[g,\chi]|^2 \overline{\delta\mathcal{G}^*_{n,m}\,\delta\mathcal{G}_{n',m'}}$$

Normalizable states require at least two insertions $(2 + 2 > 3)$.
H.H. state is not naively normalizable.

Higuchi's norm



In nongravitational limit, instead of fixing three points → divide by the volume of the conformal group.

$$(\Psi_{\rm ng}, \Psi_{\rm ng}) \propto \frac{1}{\mathrm{vol}(SO(d+1,1))} \lim_{\kappa\to 0} \langle\!\langle \delta\mathcal{G}^*_{n,m}\, \delta\mathcal{G}_{n',m'} \rangle\!\rangle$$
$$= \frac{1}{\mathrm{vol}(SO(d+1,1))} \langle \Psi_{\rm ng} | \Psi_{\rm ng} \rangle_{\rm QFT}$$

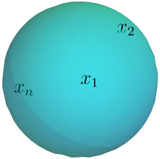
Matches Higuchi's proposal for the norm as $\kappa \to 0$ and provides gravitational corrections.

[Higuchi, 1991]

[Marolf, Morrison, 2008]

[Chandrasekaran,Longo,Penington,Witten, 2022]

Cosmological correlators



We wish to understand "cosmological correlators" on the late-time slice.

$$\langle \chi(x_1) \ldots \chi(x_n) \rangle$$

As written, expression does not commute with the constraints.

We propose interpretation as gauge-fixed operators

$$\langle\!\langle \Psi | \chi(x_1) \ldots \chi(x_n) | \Psi \rangle\!\rangle_{\mathrm{CC}} = \int |\Psi|^2 \chi(x_1) \ldots \chi(x_n) \delta(\mathrm{g.f}) \Delta'_{FP} DgD\chi$$

Symmetries of cosmological correlators

Residual gauge transformations turn into symmetries of cosmological correlators.

Translations/Dilatations:

$$\langle\langle\Psi|\chi(\lambda x_1 + v)\dots\chi(\lambda x_n + v)|\Psi\rangle\rangle_{CC} = \lambda^{-n\Delta}\langle\langle\Psi|\chi(x_1)\dots\chi(x_n)|\Psi\rangle\rangle_{CC}$$

Rotations:

$$\langle\langle\Psi|\chi(M\cdot x_1)\dots\chi(M\cdot x_n)|\Psi\rangle\rangle_{CC} = \langle\langle\Psi|\chi(x_1)\dots\chi(x_n)|\Psi\rangle\rangle_{CC}$$

SCTs relate cosmological correlators of different orders.

All states display the symmetries of the H.H. state although the precise values of cosmological correlators depend on the state.

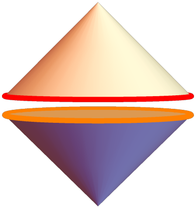
Holography of information

Gravity localizes information unusually.

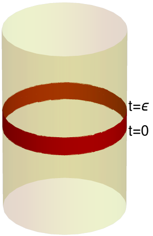
[Laddha, Prabhu, S.R., Shrivastava, 2020]

[Marolf, 2006–13]

Asymptotically flat space

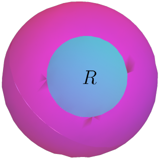


Asymptotic AdS



- Follows from analysis of gravitational constraints.
- Helps understand why gravitational theories are holographic.

Holography of information in dS



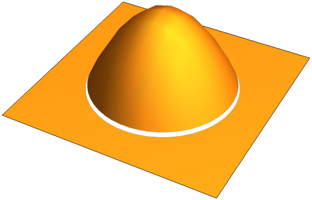
In dS, cosmological correlators in an arbitrarily small region fix cosmological correlators everywhere.

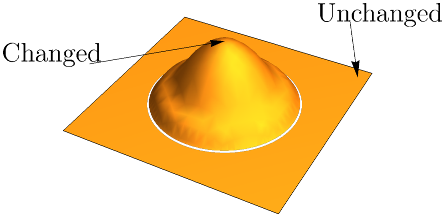
$$\langle\!\langle \Psi | \chi(\lambda x_1 + v) \ldots \chi(\lambda x_n + v) | \Psi \rangle\!\rangle_{\text{CC}} = \lambda^{-n\Delta} \langle\!\langle \langle \Psi | \chi(x_1) \ldots \chi(x_n) | \Psi \rangle\!\rangle_{\text{CC}}$$

Holography of information and cosmological correlators

$$\langle\!\langle \Psi_1 | \chi(x_1) \ldots \chi(x_n) | \Psi_1 \rangle\!\rangle_{\text{CC}} = \langle\!\langle \Psi_2 | \chi(x_1) \ldots \chi(x_n) | \Psi_2 \rangle\!\rangle_{\text{CC}} \forall x_i \in \mathcal{R},$$
$$\implies \langle\!\langle \chi(x_1) \ldots \chi(x_n) | \Psi_1 \rangle\!\rangle_{\text{CC}} = \langle\!\langle \Psi_2 | \chi(x_1) \ldots \chi(x_n) | \Psi_2 \rangle\!\rangle_{\text{CC}} \forall x_i,$$

In sharp contrast to QFT.





Nongravitational limit

Holography of information persists in the nongravitational limit.

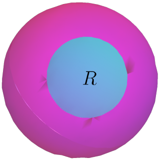
if $\forall x_i \in \mathcal{R}$,

$\langle\!\langle \Psi_{\text{ng},1} | \chi(x_1) \ldots \chi(x_n) | \Psi_{\text{ng},1} \rangle\!\rangle_{\text{CC}} = \langle\!\langle \Psi_{\text{ng},2} | \chi(x_1) \ldots \chi(x_n) | \Psi_{\text{ng},2} \rangle\!\rangle_{\text{CC}}$

then $\forall x_i$,

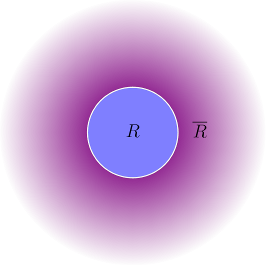
$\langle\!\langle \Psi_{\text{ng},1} | \chi(x_1) \ldots \chi(x_n) | \Psi_{\text{ng},1} \rangle\!\rangle_{\text{CC}} = \langle\!\langle \Psi_{\text{ng},2} | \chi(x_1) \ldots \chi(x_n) | \Psi_{\text{ng},2} \rangle\!\rangle_{\text{CC}}$,

$$|\Psi_{\text{ng}}\rangle = \int dx_i f(x_1, \ldots x_n) \chi(x_1) \ldots \chi(x_n) |0\rangle$$

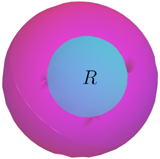


Holography of information

AdS and flat space



dS



The complement of a bounded region has all information about the state.

Conclusion

- **Hilbert space:** Solutions of WDW-eqn (in the large-volume limit) are of the form $e^{iS}Z[g,\chi]$, where $|Z[g,\chi]|^2$ is a diff and Weyl-invariant functional.
- All allowed states are of this form, not just the vacuum. (Vacuum itself does not appear normalizable.)
- **Symmetries.** Cosmological correlators, after gauge-fixing, are covariant under scaling, rotations, translations in all states. SCTs relate different cosmological correlators.
- **Holography of information:** Specifying cosmological correlators in an arbitrarily small region specifies them everywhere. Sharp contrast with QFT.

Thank you

Appendix

Gauge-fixing for cosmological correlators

$$\langle\!\langle \Psi_1 | \chi(x_1) \ldots \chi(x_n) | \Psi_2 \rangle\!\rangle_{CC} = \int \Psi_1^* \Psi_2 \chi(x_1) \ldots \chi(x_n) \delta(\mathrm{g.f}) \Delta'_{FP} DgD\chi$$

gives unambiguous prescription for the matrix elements.

$\exists$ gauge invariant operator with the same matrix elements.

When $\kappa \to 0$,

$$\hat{C} = \int [dU] U^\dagger \chi(x_1) \ldots \chi(x_n) U$$

independent of gauge choice.

At nonzero κ, gauge choice matters. Gauge-fixing $\longrightarrow$ setting our reference frame as observers.

Weyl transformation of variables

We are interested in $Z[g, \chi]$ in the regime where

$$g_{ij}^{\text{phys}} = \frac{4\omega^2}{(1+|x|^2)^2}(\delta_{ij} + \kappa h_{ij}),$$

with large ω

Since $|Z[g, \chi]|^2$ is diff-and-Weyl invariant, transform to frame where

$$g_{ij} = \delta_{ij} + \kappa h_{ij}$$

Explicit constraints

Explicitly,

$$\mathcal{H} = 2\kappa^2 g^{-1}\left(g_{ik}g_{jl}\pi^{kl}\pi^{ij} - \frac{1}{d-1}(g_{ij}\pi^{ij})^2\right) - \frac{1}{2\kappa^2}(R - 2\Lambda)$$
$$+ \mathcal{H}_{\text{matter}} + \mathcal{H}_{\text{int}},$$
$$\mathcal{H}_i = -2g_{ij}D_k\frac{\pi^{jk}}{\sqrt{g}} + \mathcal{H}_i^{\text{matter}},$$

These constraints are equivalent to the Einstein equations. But we are imposing

$$\mathcal{H}\Psi = \mathcal{H}_i\Psi = 0$$

Different from solving

$$G_{\mu\nu} = 8\pi G\langle T_{\mu\nu}\rangle$$