Nonperturbative Anomalous Thresholds

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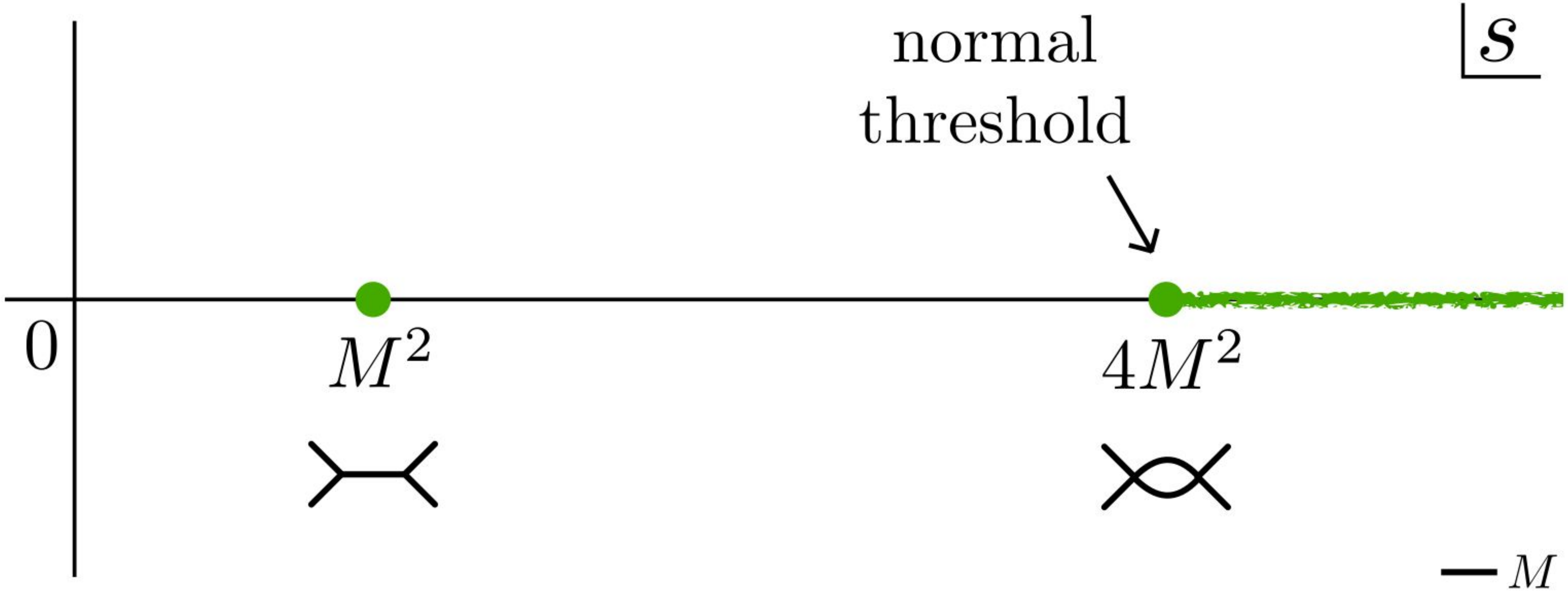
What are anomalous thresholds?

Consider $2 \to 2$ scattering amplitude: $T_{MM \to MM}(s,t)$

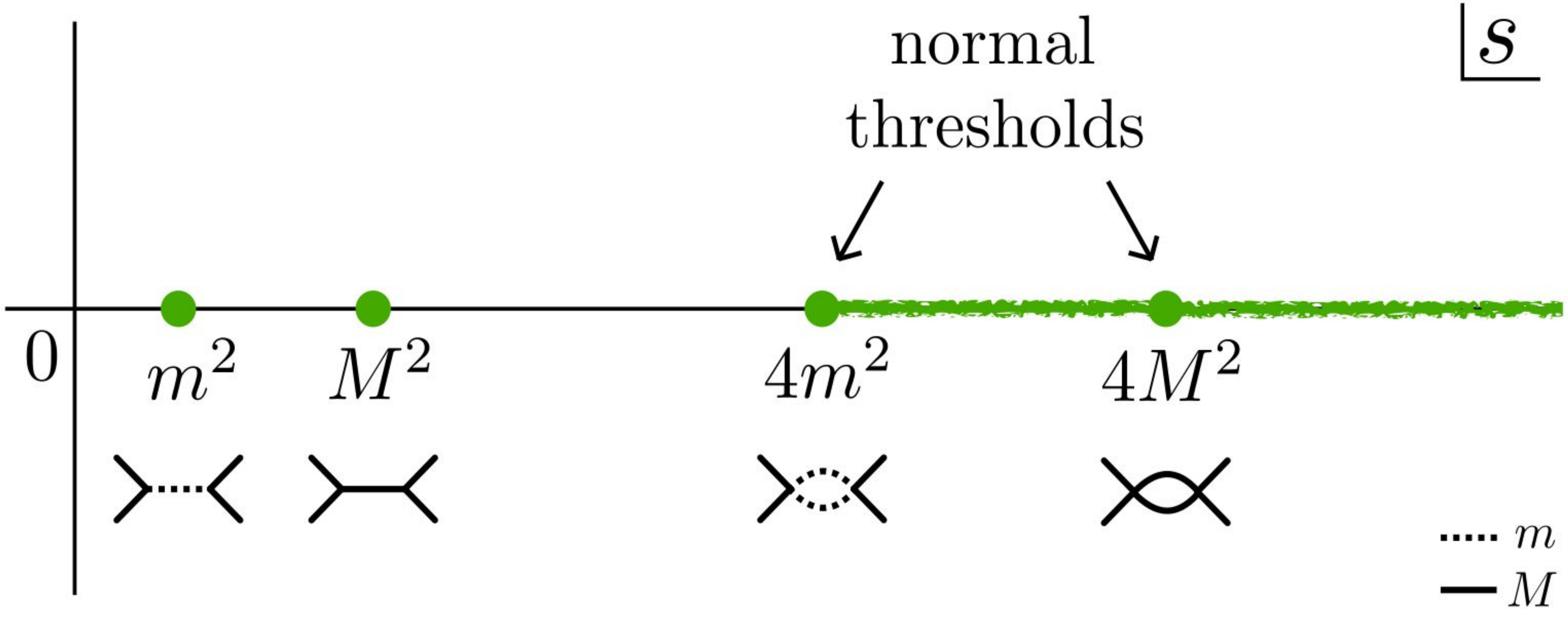


Normal thresholds follow directly from Unitarity:

$$\mathrm{Im} T_{2\to 2}(s,t) = \sum\!\!\!\!\!\!\int_n T_{2\to n} T^\dagger_{2\to n}$$

$$= |T_{2\to 1}|^2 \delta(s - M^2) + \int |T_{2\to 2}|^2 \Theta(s - 4M^2) + \cdots$$

Add another particle $m < M$ to the spectrum:



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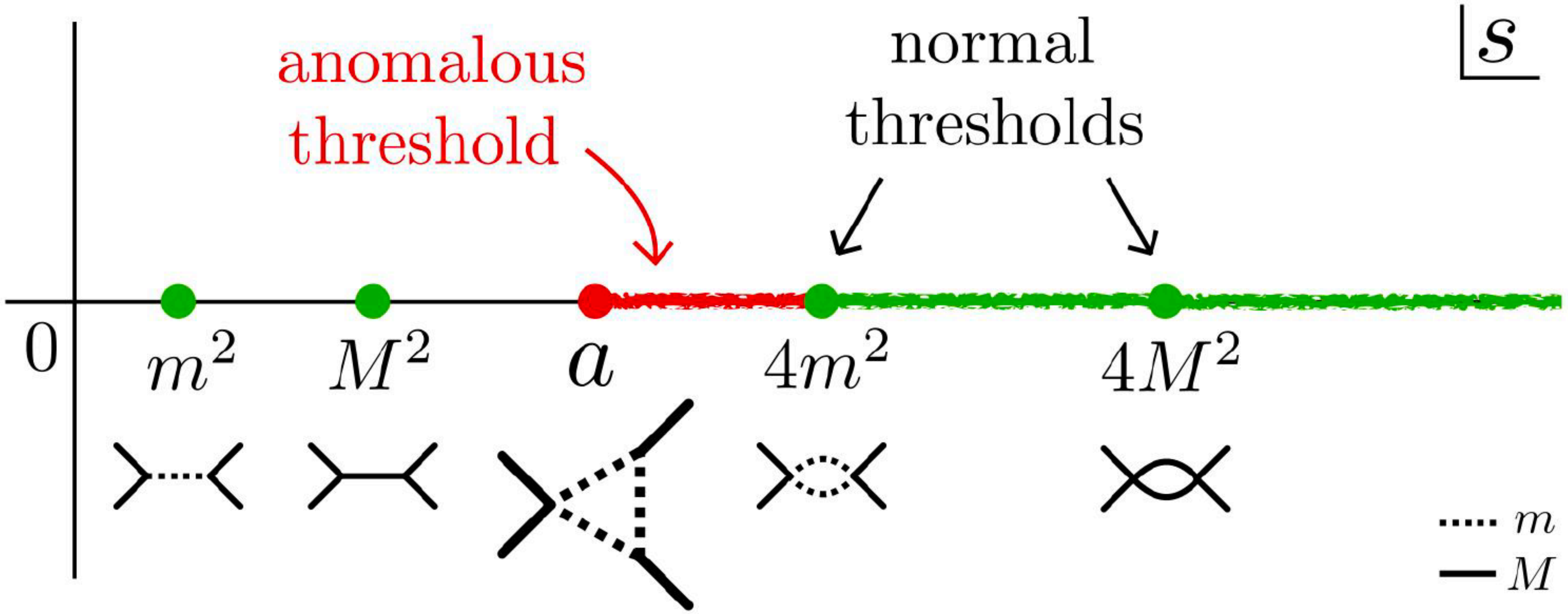
For $M > \sqrt{2}m$ an anomalous threshold appears at $a = 4M^2 - M^4/m^2$



$$\mathrm{Im}T(s,t)|_{\mathrm{anomalous}} = ?$$

Why is this problem important?

S-matrix bootstrap state of the art

- $2 \to 2$ scattering:
  - $mm \to mm$ (1960s-70s, 2016+)
  - $MM \to MM$; $MM \to mm$ $\quad M < \sqrt{2}m$ [Homrich,Penedones,Toledo, van Rees, Vieira] (2019)
  - $MM \to MM$; $MM \to mm$ $\quad M > \sqrt{2}m$
- $2 \to 3$ scattering
- $2 \to 4$ scattering
- $2 \to 5$ scattering
- $2 \to 6$ scattering
- $2 \to 7$ scattering
- $2 \to 8$ scattering
- ...

Anomalous Thresholds!

S-matrix principles/constraints

- Analyticity in s and t
- Crossing symmetry
- Unitarity

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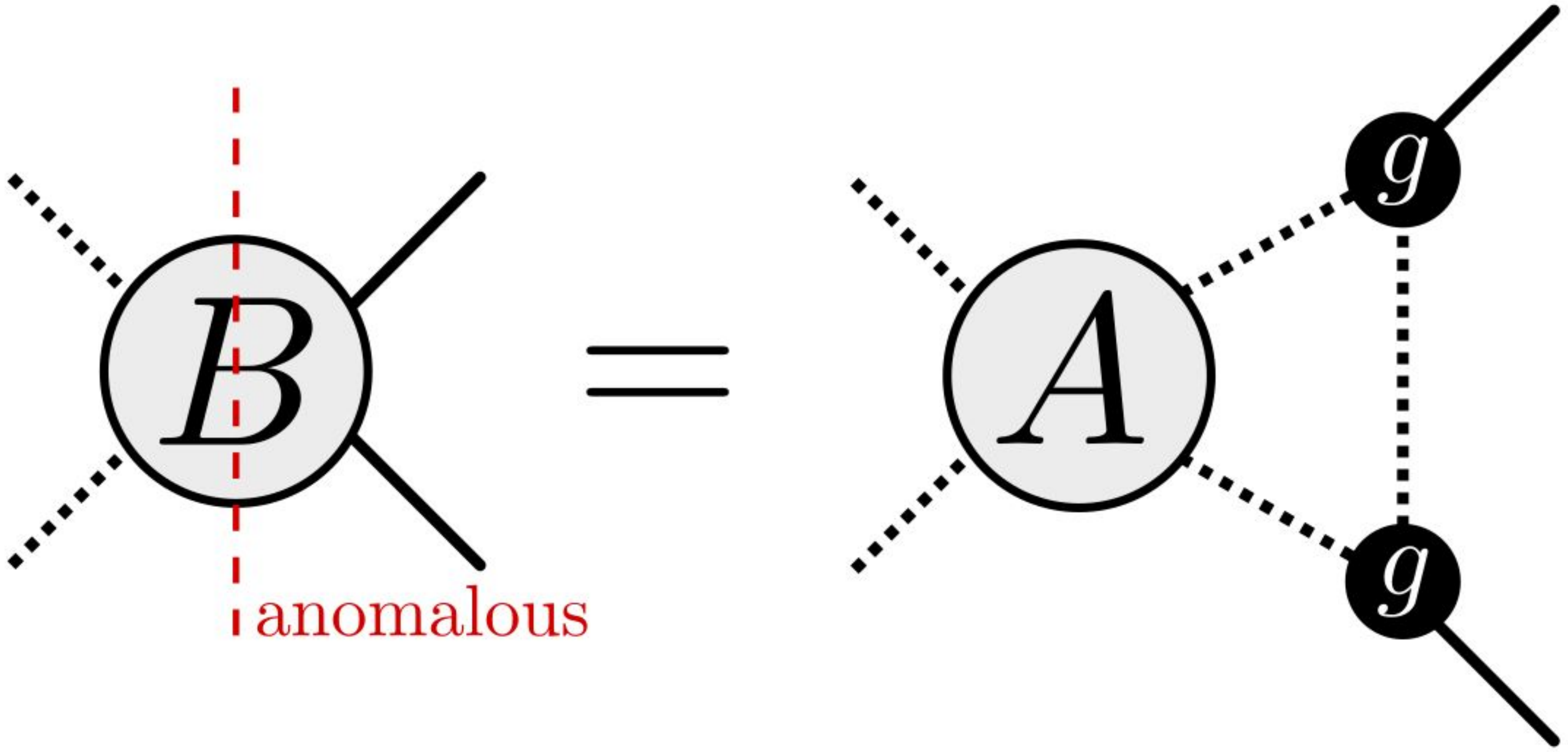
- Analyticity in M

Results

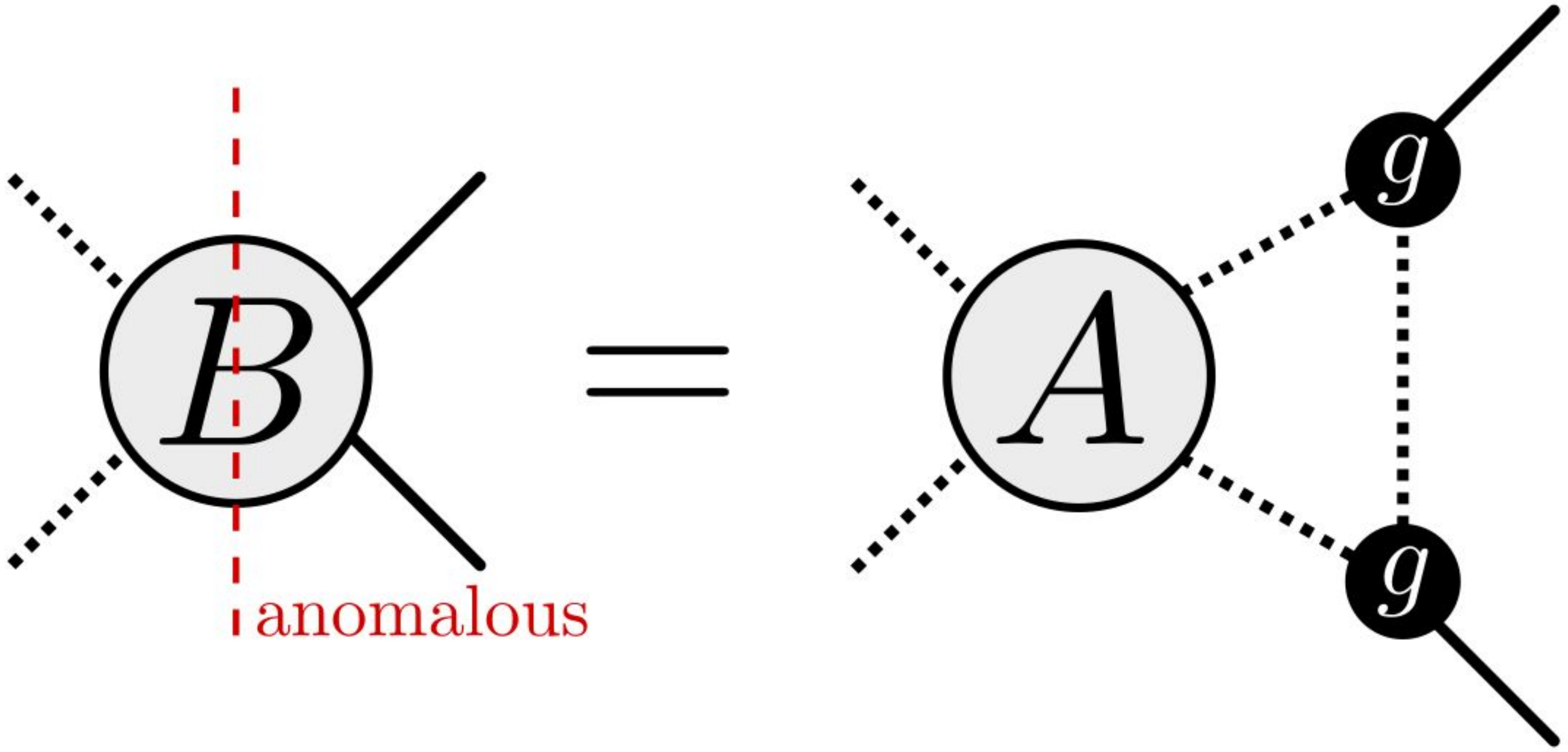
Results: $MM \to mm$



Results: $MM \to mm$

$$B(s \to a) = -\frac{g^2 A(a)}{s-a}, \qquad d = 2$$

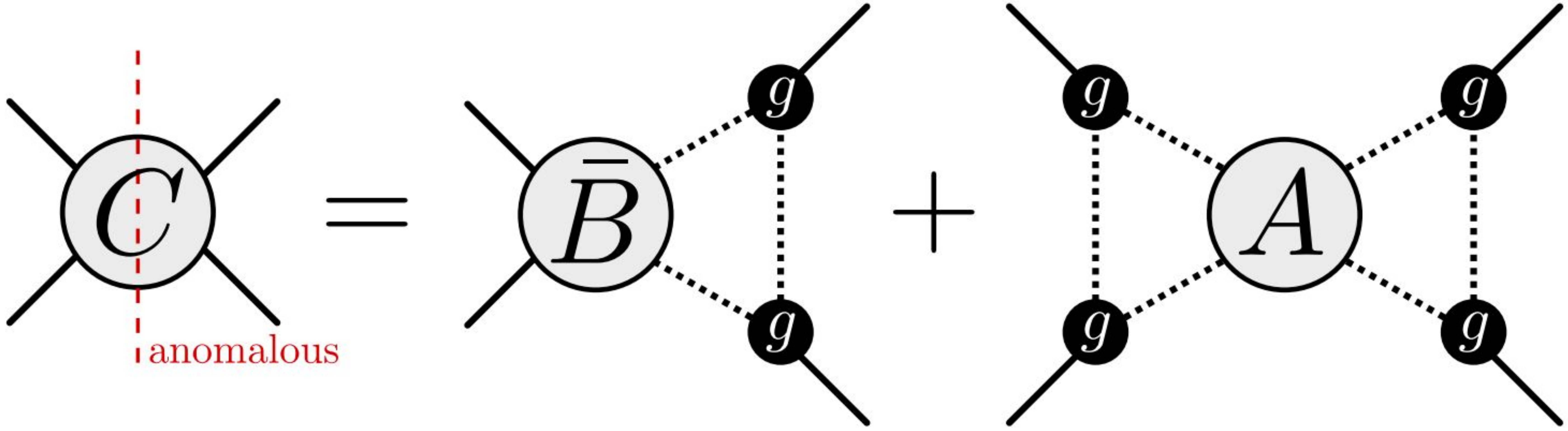
$$\mathrm{Im}\, B(s,t) = -\frac{g^2 \int \mathcal{K}(t, m^2, t') A(s,t') dt'}{\sqrt{s(4M^2 - s)}} \Theta(a \leq s < 4m^2), \qquad d = 4$$



Results: $MM \to MM \quad (d=2)$

$$C(s\to a) = -4\mathcal{N}\varrho(a)\,\frac{g^2\,\bar{B}(a)}{s-a} + 2\mathcal{N}^2\varrho(a)\,\frac{g^4\,S(a)}{(s-a)^2}$$

$$+\,2\mathcal{N}^2\,\frac{g^4}{s-a}\left[\frac{d\varrho(a)}{ds}S(a) - \varrho(a)\frac{dS(a)}{ds}\,\frac{1-S(a)}{1+S(a)}\right]$$



Results: $MM \to MM \quad (d=4)$

$$
\begin{aligned}
\operatorname{Disc} C_J(s) = -\frac{P_J\left(z_a(s)\right)}{4\sqrt{s(4M^2-s)}}\Bigg[& g^2 \bar{B}_J(s) \\
& + \frac{g^4}{8\pi}\int_{-\infty}^{a} \frac{\tilde{P}_J(s')}{s'-s}\,\frac{1+\varrho(s)A_J(s)}{1+\varrho(s')A_J(s')}ds' \\
& - \frac{g^4}{4\pi}\int_{a}^{4m^2} \frac{\tilde{P}_J(s')\varrho(s')A_J(s')}{s'-s}\,\frac{1+\varrho(s)A_J(s)}{1+\varrho(s')A_J(s')}ds'\Bigg]
\end{aligned}
$$