Self-Dual Black Holes in Celestial Holography

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Strings Gong Show

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Based on: arXiv: 2302.06661 w/ A. Guevara, E. Himwich, A. Strominger

Overview

Goal: Extend previous work on

Classical limit of scattering amplitudes $\leftrightarrow$ Black hole spacetimes

to give a notion of Black Holes in Celestial Holography

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Approach: Consider (2, 2) signature linearized, self-dual Kerr-Taub-NUT

Connect to amplitudes and utilize useful celestial holography results. Also connections to twistor theory and more!

Self-dual/anti-self-dual correlates to $+/-$ helicity (in amplitudes picture) and holomorphic/antiholomorphic limits in celestial holography

The existence of Kerr-Schild form of the metric means linearized results could extend to non-linear results

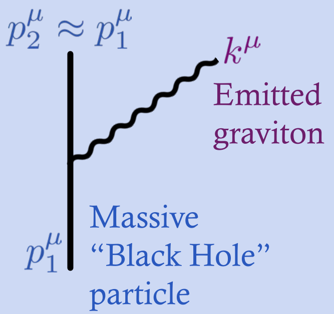
Goal: Extend Classical limit of scattering amplitudes $\leftrightarrow$ Black hole spacetimes

to afford a realization of Black Holes in Celestial Holography

Goal: Extend Classical limit of scattering amplitudes ↔ Black hole spacetimes to afford a realization of Black Holes in Celestial Holography

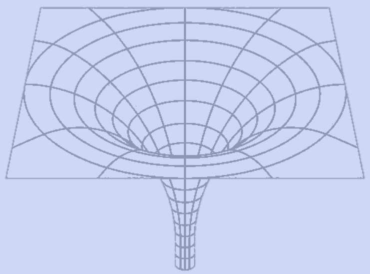
Amplitudes ↔ Spacetimes

- In (2, 2) signature, $\mathcal{O}(G)$ part of a stationary metric can be obtained *directly* from the classical limit of on-shell scattering amplitudes
  [Monteiro, O'Connell, Veiga, Sergola '20; Crawley, Miller, Guevara, Strominger '21; Monteiro, Nagy, O'Connell, Veiga, Sergola '21]



3-point amplitude

Directly map to



Linearized Black Hole spacetime

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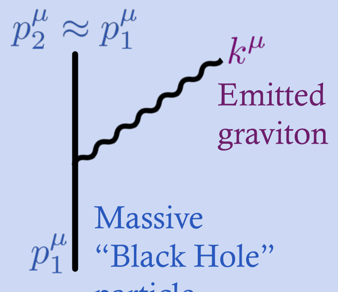

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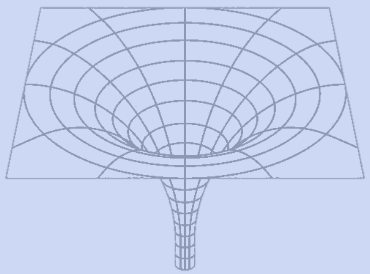

Linearized Black Hole spacetime

Celestial Holography

- Recast 4d scattering amplitudes from

  $e^{i\omega \hat{k}(z,\bar{z})\cdot x}$ plane waves (diagonalizing translations) $\xrightarrow[\text{Mellin Transform}]{\omega \to \Delta}$ $\varphi_\Delta(x; z, \bar{z})$ conformal primary wavefunctions (diagonalizing boosts)

- 4d Lorentz = 2d conformal symmetry, so amplitudes transform as correlators on celestial boundary. Subleading soft graviton theorem enhances this to Virasoro.
- In (2, 2) signature, celestial sphere $\to$ (1, 1) celestial torus, parameterized by $(z, \bar{z})$
  [Atanasov, Ball, Melton, Raclariu, Strominger '21]

Black Holes in a Conformal Primary Basis

Consider linearized self-dual Kerr-Taub-NUT Mass (M), NUT charge (N), spin (a)

$$ds^2 = \left(\eta_{\mu\nu} + h_{\mu\nu} + \mathcal{O}(G^2)\right) dx^\mu\, dx^\nu$$

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$h_{\mu\nu}(x)$ is related to a graviton emission three-point amplitude:

[Crawley, Miller, Guevara, Strominger '21]

$$h_{\mu\nu}(x) \propto G \int d\begin{pmatrix}\text{on-shell} \\ \text{k space}\end{pmatrix} \varepsilon^-_{\mu\nu} e^{-k\cdot x} \, \mathcal{M}_3(k)$$

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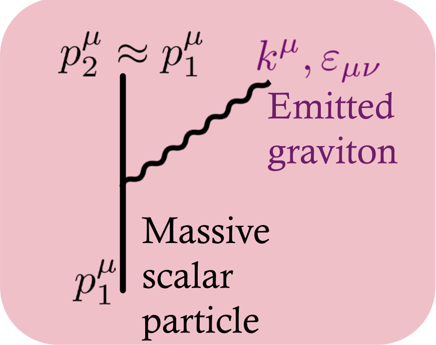

$$\mathcal{M}_3(k) \propto (M+N) e^{-k\cdot a} \delta(k^0)$$

[Emond, Huang, Kol, Moynihan, O'Connell '20, see also Arkani-Hamed, Huang, Huang '17, Guevara, Ochirov, Vines '18; Huang, Kol, O'Connell '20]

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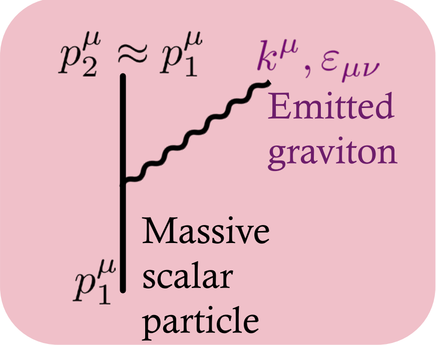
Integrate over on-shell graviton momentum

$k^\mu = \omega \hat{k}^\mu(z,\bar{z}) = \omega(z+\bar{z}, 1+z\bar{z}, z-\bar{z}, 1-z\bar{z}),$



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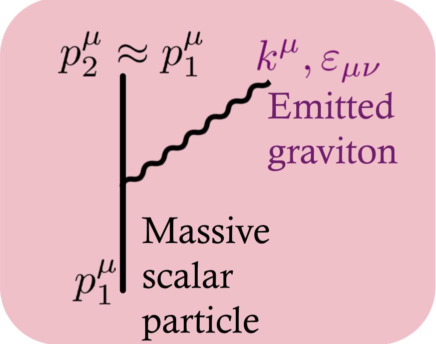

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This can be recast into a conformal primary basis with $\Delta \in \mathbb{Z}$:

$$h_{\mu\nu}(x) \propto G \sum_{\Delta=0}^{\infty} \int dz d\bar{z} \, \varepsilon^-_{\mu\nu}(z,\bar{z}) \, \varphi_{\Delta+1}(x; z, \bar{z}) \, (M+N) \frac{a^\Delta}{\Delta!} \delta(\hat{k}^0),$$

Conformal primary wavefunction

Related to $\mathcal{M}_3(k)$

Black Hole Construction on the Celestial Torus

$$h_{\mu\nu}(x) \propto G \sum_{\Delta=0}^{\infty} \int dz d\bar{z}\, \varepsilon^{-}_{\mu\nu}(z,\bar{z})\, \varphi_{\Delta+1}(x;z,\bar{z})\, (M+N) \frac{a^{\Delta}}{\Delta!} \delta(z+\bar{z})\,,$$

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Specialize to gravitational potential (0,0 component)

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Black hole perturbation can be expressed in a local ($\varphi_{\Delta}(x;z,\bar{z})$) or global ($\Phi^{\Delta}_{m,n}(x)$) celestial basis!

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Now, promote $h_{00}(x)$ to an operator – $\hat{h}_{00}(x) \propto G \sum_{\Delta=1}^{\infty} \sum_{m,n} H^{2-\Delta}_{m,n} \Phi^{\Delta}_{m,n}(x)$

Quantum state on the celestial torus

Goal: Find $|BH\rangle$ such that $\langle BH|\hat{h}_{00}|BH\rangle$ (equivalently $\langle BH|H^{2-\Delta}_{m,n}|BH\rangle$) reproduces the classical metric

Quantum Construction on the Celestial Torus

Equipped with $\hat{h}_{00}(x) \propto G \sum_{\Delta=1}^{\infty} \sum_{m,n} H_{m,n}^{2-\Delta} \Phi_{m,n}^{\Delta}(x)$:

Quantum Construction on the Celestial Torus

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- The mode coefficients $H_{m,n}^{\Delta}$ are soft graviton modes $\Delta = 1$, $\Delta = 0$, $\Delta = \ldots$

  $\Delta = 1$: Leading soft graviton

  $\Delta = 0$: Subleading soft graviton

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  Leading soft graviton ($\Delta = 1$); Subleading soft graviton ($\Delta = 0$)

- Define $|BH\rangle \equiv \exp\left[i\sqrt{G}\, M \sum_{\Delta=1}^{\infty} \frac{a^{\Delta-1}}{(\Delta-1)!} \sum_{j} \mathcal{G}^{\Delta}_{j,j} \right] |0\rangle,$

  Goldstone modes, where $\mathcal{G}^{\Delta}_{m,n}$ is canonically paired with $H^{2-\Delta}_{-m,-n}$

  See also [Freidel, Pranzetti, Raclariu '22; Nguyen, Salzer '20; Donnay, Pasterski, Puhm '20 and '22; Pasterski, Puhm, Trevisani '21]

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- This construction yields $\langle BH|\hat{h}_{00}|BH\rangle = h_{00}$

Quantum expectation value reproduces the classical metric!

Summary

- Three-point graviton emission amplitudes and linearized black hole spacetimes can be directly connected in (2, 2) signature.
- Classically, we can use this connection to write a linearized black hole perturbation in either a local or global conformal primary basis.
- In the 2d quantum/celestial theory, the metric is promoted to an operator, constructed from a tower of soft graviton operators. A coherent state associated to the black hole can be constructed out of 2d Goldstone operators, so that the metric expectation value in this state reproduces the classical result.
- Out of time, but see paper for:
  - Connections to $w_{1+\infty}$ charges
  - Connections to Wilson lines
  - Checks and agreement with results on celestial holography in curved backgrounds

Thank you!