Chaos and the Reparametrization Mode on the AdS_2 string

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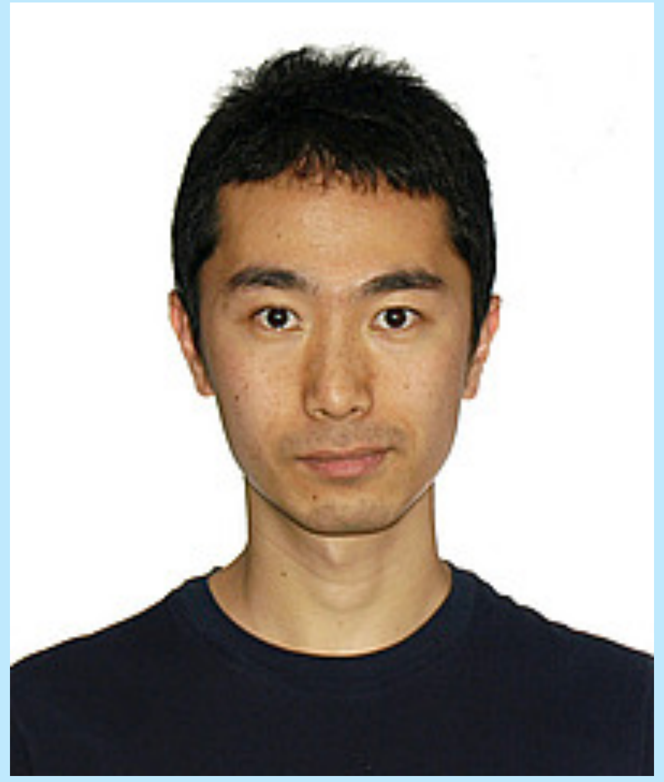
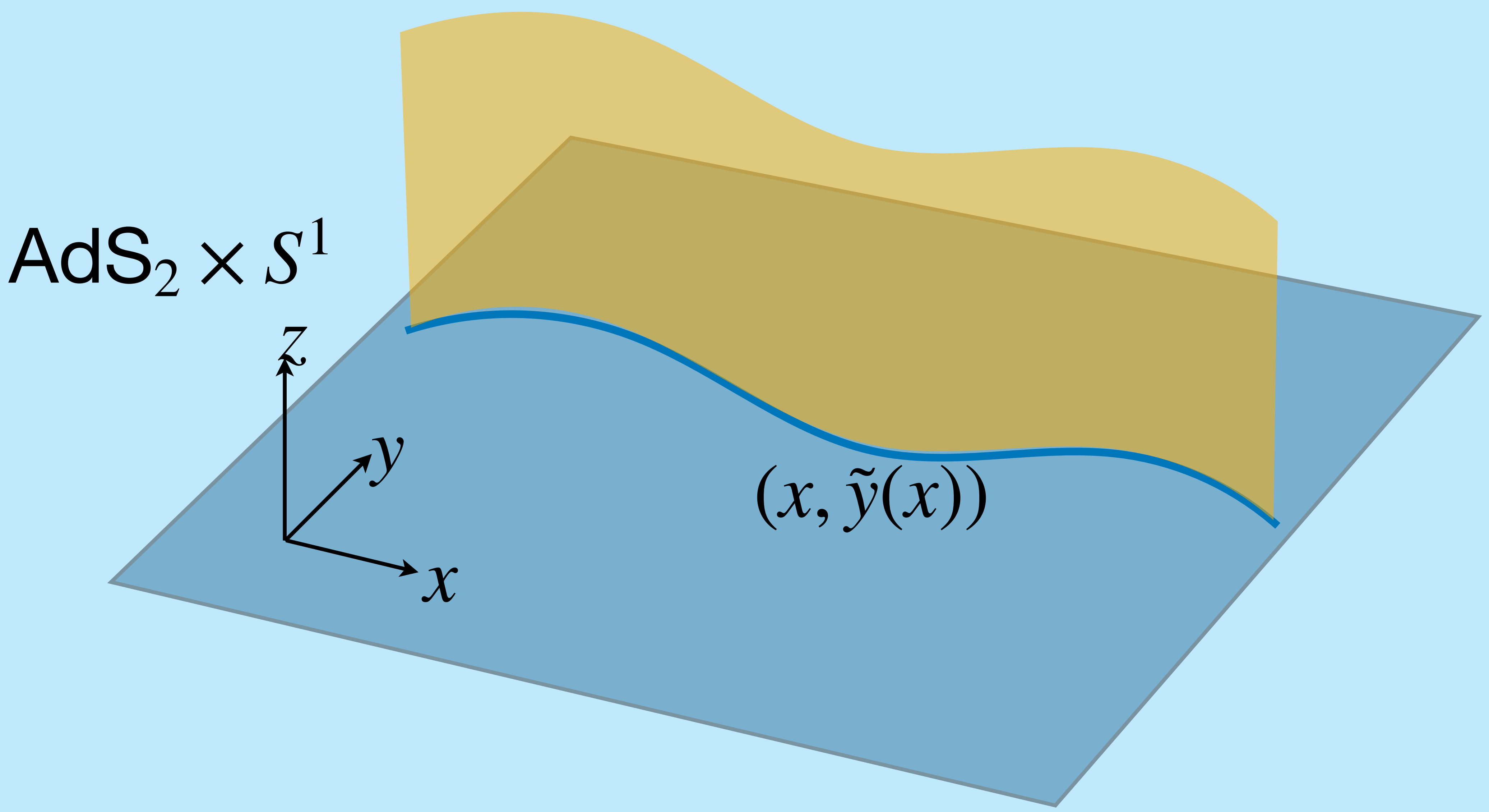
Strings 2023



based on arXiv:2212.14842 and work in progress with Simone Giombi, Shota Komatsu, Jieru Shan

Set-up: boundary correlators on the AdS_2 string



$$ds^2 = \frac{dx^2 + dz^2}{z^2} + dy^2$$

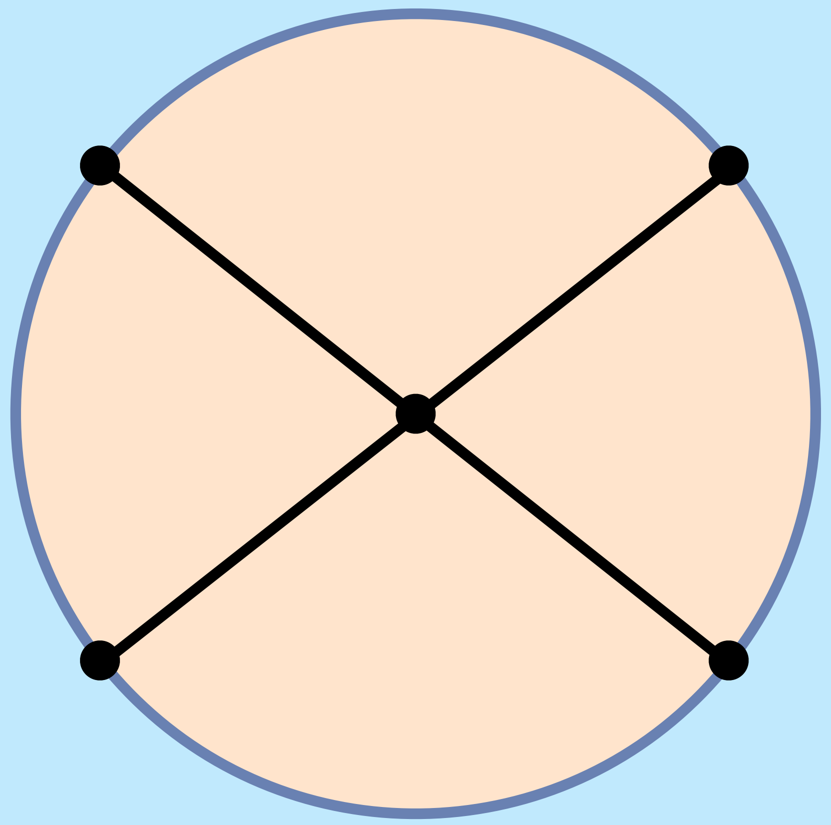
$$g_s = 0,\ T_s \gg 1: \quad Z[\tilde{y}] \approx e^{-S_{\mathrm{cl}}[\tilde{y}]}$$

$$\langle y(x_1)\ldots y(x_n)\rangle = \frac{1}{Z[0]} \frac{\delta^n Z[\tilde{y}]}{\delta\tilde{y}(x_1)\ldots\tilde{y}(x_n)}$$

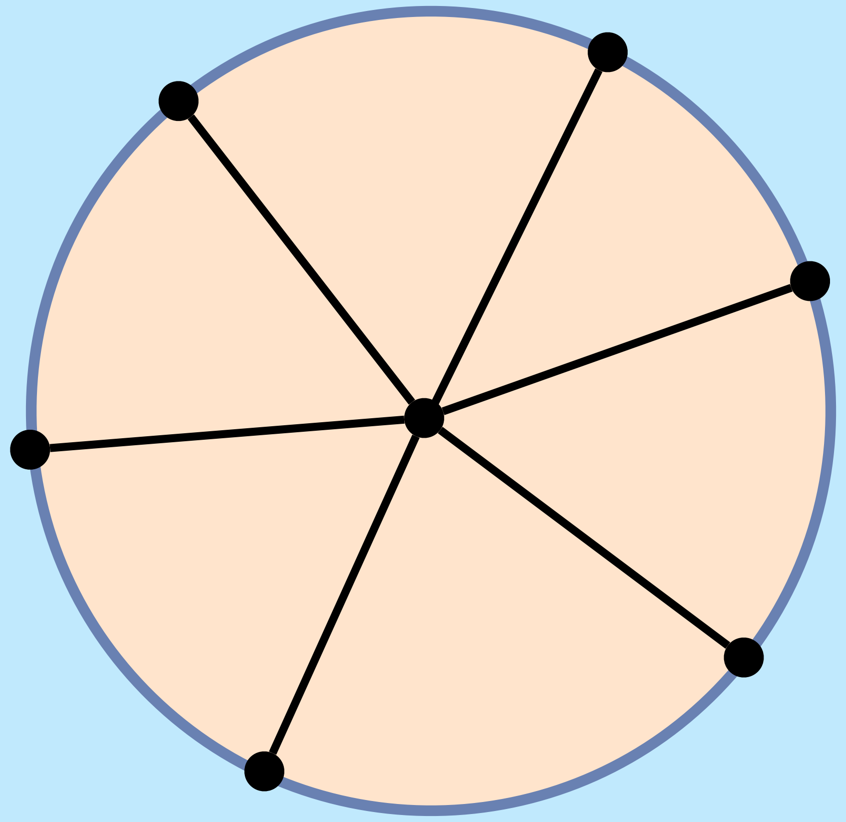
Static gauge

[Giombi, Roiban, Tseytlin '17]

$$\langle y(x_1)\ldots y(x_4)\rangle_{\rm c} =$$



$$\langle y(x_1)\ldots y(x_6)\rangle_{\rm c} =$$



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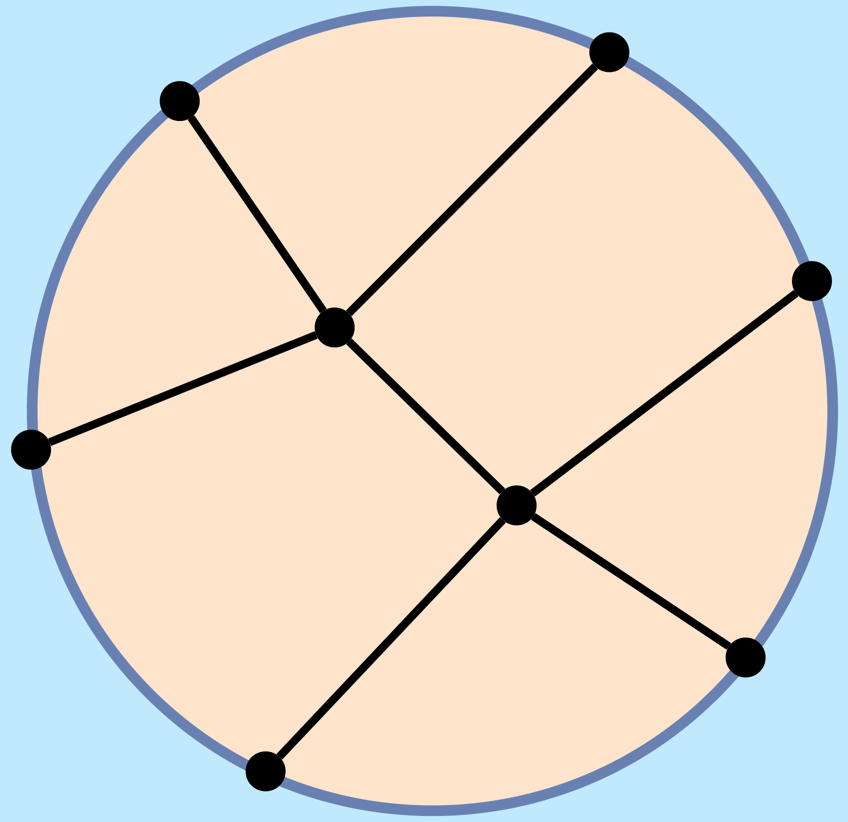

Conformal gauge

[See also Polyakov, Rychkov '00 '01; Rychkov '02; Ambjorn, Makeenko '12]

$$S = S_L + S_T, \qquad S_L[x,z] = \frac{T_s}{2}\int d^2\sigma \frac{\partial_\alpha z \partial^\alpha z + \partial_\alpha x \partial^\alpha x}{z^2}, \quad S_T[y] = \frac{T_s}{2}\int d^2\sigma \partial_\alpha y \partial^\alpha y$$

Here, $\sigma^\alpha = (s,t)$ and the BCs are: $\quad x(0,t) = \alpha(t), \quad z(0,t) = 0, \quad y(0,t) = \tilde{y}(\alpha(t))$

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Classically: $$Z[\tilde{y}] \approx e^{-S_L[\alpha]-S_T[\tilde{y},\alpha]}\Big|_{\text{Virasoro}} = e^{-S_L[\alpha]-S_T[\tilde{y},\alpha]}\Big|_{\text{extremize }\alpha}$$

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On-shell transverse action: $\quad S_T[\tilde{y},\alpha] = -\frac{T_s}{2\pi}\int dt_1 dt_2 \frac{\tilde{y}(\alpha(t_1))\tilde{y}(\alpha(t_2))}{t_{12}^2}$

On-shell longitudinal action ($\alpha(t) = t + \epsilon(t)$):

$$S_L[\alpha] = \frac{6T_s}{\pi}\int dt_1 dt_2 \frac{\epsilon(t_1)\epsilon(t_2)}{t_{12}^4} - \frac{12T_s}{\pi}\int dt_1 dt_2 \frac{\epsilon(t_1)^2\epsilon(t_2)}{t_{12}^5} + O(\epsilon^4)$$

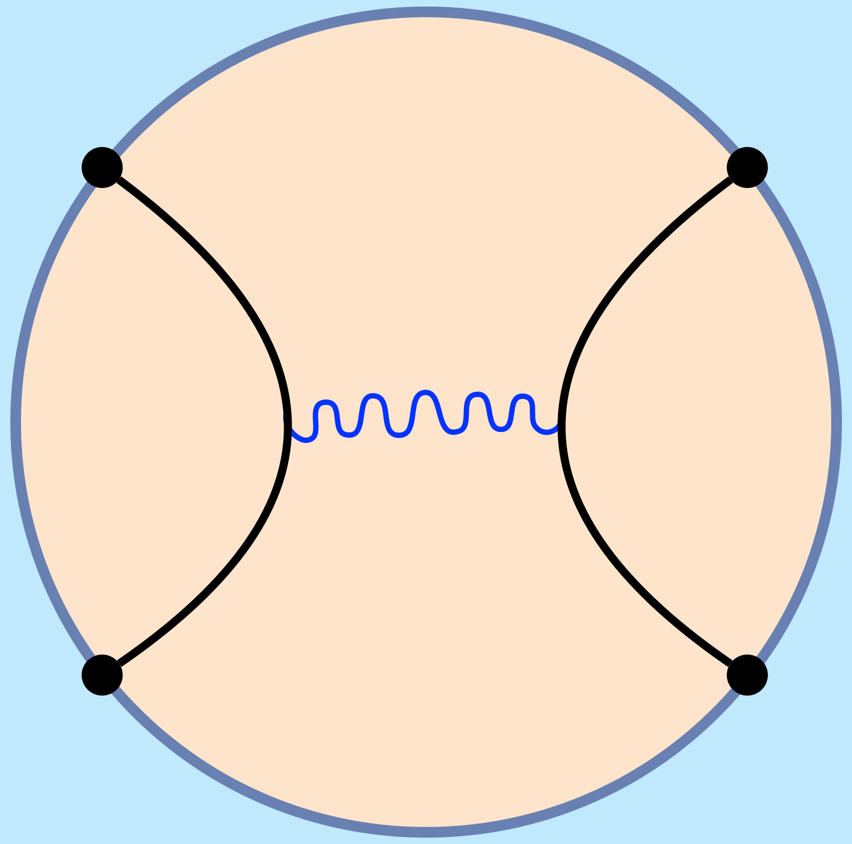
Conformal gauge

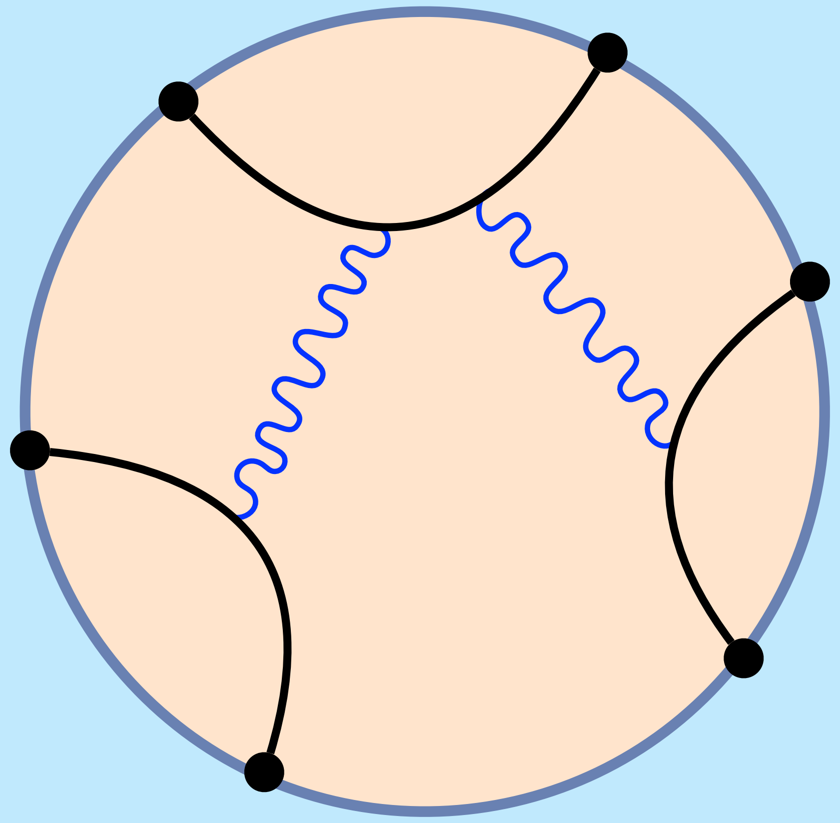
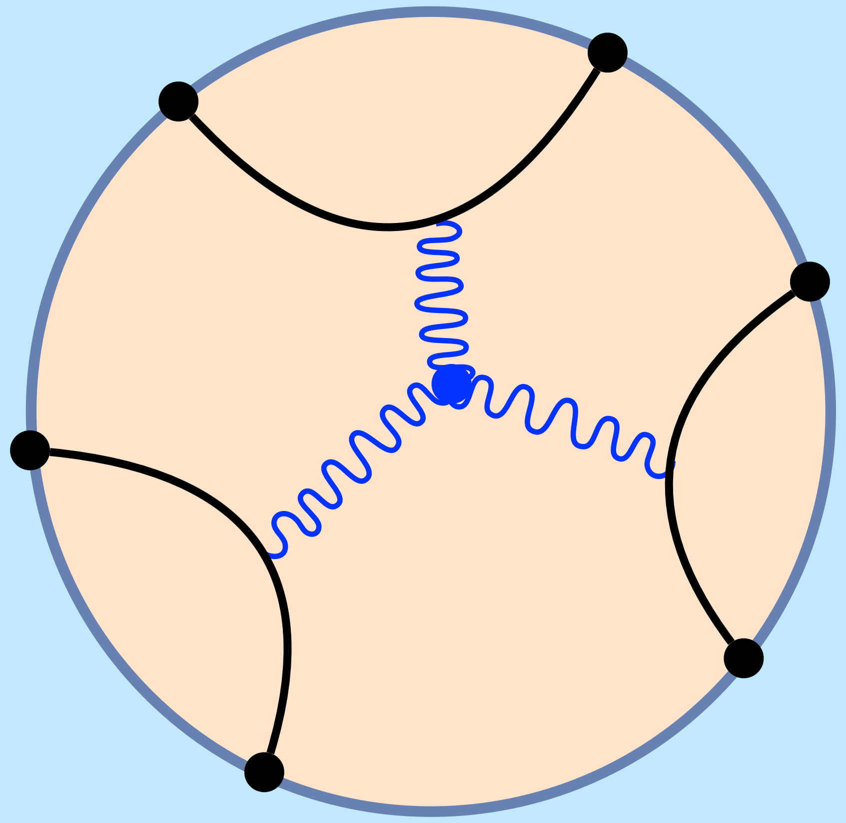
We write:

$$Z[\tilde{y}] = \int \mathcal{D}\alpha e^{-S_L[\alpha]+S_T[\alpha,\tilde{y}]}$$

Taking derivatives w.r.t. $\tilde{y}$ pulls down $$B(x_1, x_2) = \frac{\dot{\alpha}^{-1}(x_1)\dot{\alpha}^{-1}(x_2)}{(\alpha^{-1}(x_1) - \alpha^{-1}(x_2))^2}.$$

Meanwhile, the propagator for ϵ is $$\langle \epsilon(x)\epsilon(0)\rangle = a + bx^2 + \frac{1}{8\pi T_s}x^2 \log x^2.$$

$\langle y(x_1)\ldots y(x_4)\rangle_c =$ 

$\langle y(x_1)\ldots y(x_6)\rangle_c =$  + 

Conformal gauge

- Thus, the conformal gauge provides a viable approach to doing perturbative computations of the string boundary correlators (at least at tree level)

- However, our original motivation for studying the conformal gauge was to better understand the out-of-time-order correlator (OTOC) on the AdS_2 string

OTOC on the AdS_2 string

- OTOCs are simple diagnostics of quantum chaos. [Larkin, Ovchinnikov, Kitaev, Shenker, Stanford, Maldacena, …]

- For the string four-point function, we set $\chi(t) = \dfrac{2}{1 - i\sinh t}$ (and $\beta = 2\pi$) and find:

$$\frac{\langle yy(t)yy(t)\rangle}{\langle yy\rangle\langle yy\rangle} = 1 - \frac{e^t}{4T_s} + \dots$$

This saturates the chaos bound (!) [de Boer, Llabres, Pedraza, Vegh '17; Murata '17]

- $\langle y_1 y_2 y_3 y_4\rangle$ was bootstrapped to three-loops [Ferrero Meneghelli '21] and we find:

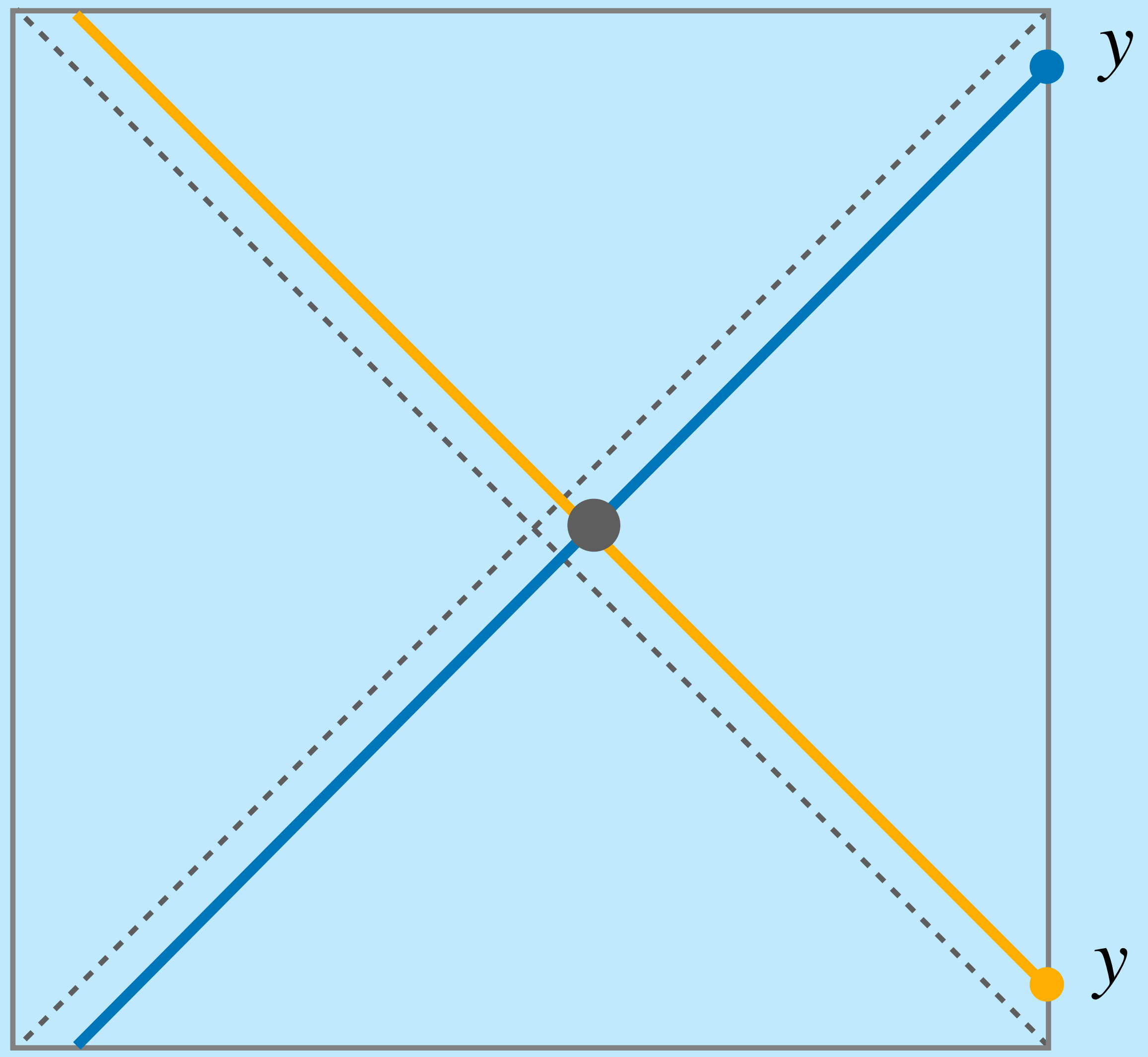
$$\frac{\langle yy(t)yy(t)\rangle}{\langle yy\rangle\langle yy\rangle} = 1 - \frac{e^t}{4T_s} + \frac{9e^{2t}}{128T_s^2} - \frac{3e^{3t}}{128T_s^3} + \frac{75e^{4t}}{8192T_s^4} + \dots$$

$$= \frac{1}{\kappa^2}U(2,1,\kappa^{-1}), \qquad \text{where } \kappa = \frac{e^t}{16T_s}.$$

This is the same as in JT gravity (!)

[Maldacena, Stanford, Yang '16; Lam, Mertens, Turiaci, Verlinde '18]

OTOC from scattering or "eikonal" resummation



[Shenker, Stanford '14; de Boer, Llabres, Pedraza, Vegh '17]

$$\langle y_1 y_3 y_2 y_4 \rangle = \int dp^u dp^v \underbrace{\Psi(p^u, t_1)^* \Phi(p^v, t_3)^*}_{\text{out state}} \underbrace{e^{i\ell_s^2 p^u p^v}}_{\text{S-phase}} \underbrace{\Psi(p^u, t_2)\Phi(p^v, t_4)}_{\text{in state}}$$

$$= \frac{1}{\kappa^2} U(2,1,\kappa^{-1})$$

Here, $e^{i\ell_s^2 p^u p^v}$ is the scattering phase on the long flat string [Dubovsky, Flauger, Gorbenko '12]

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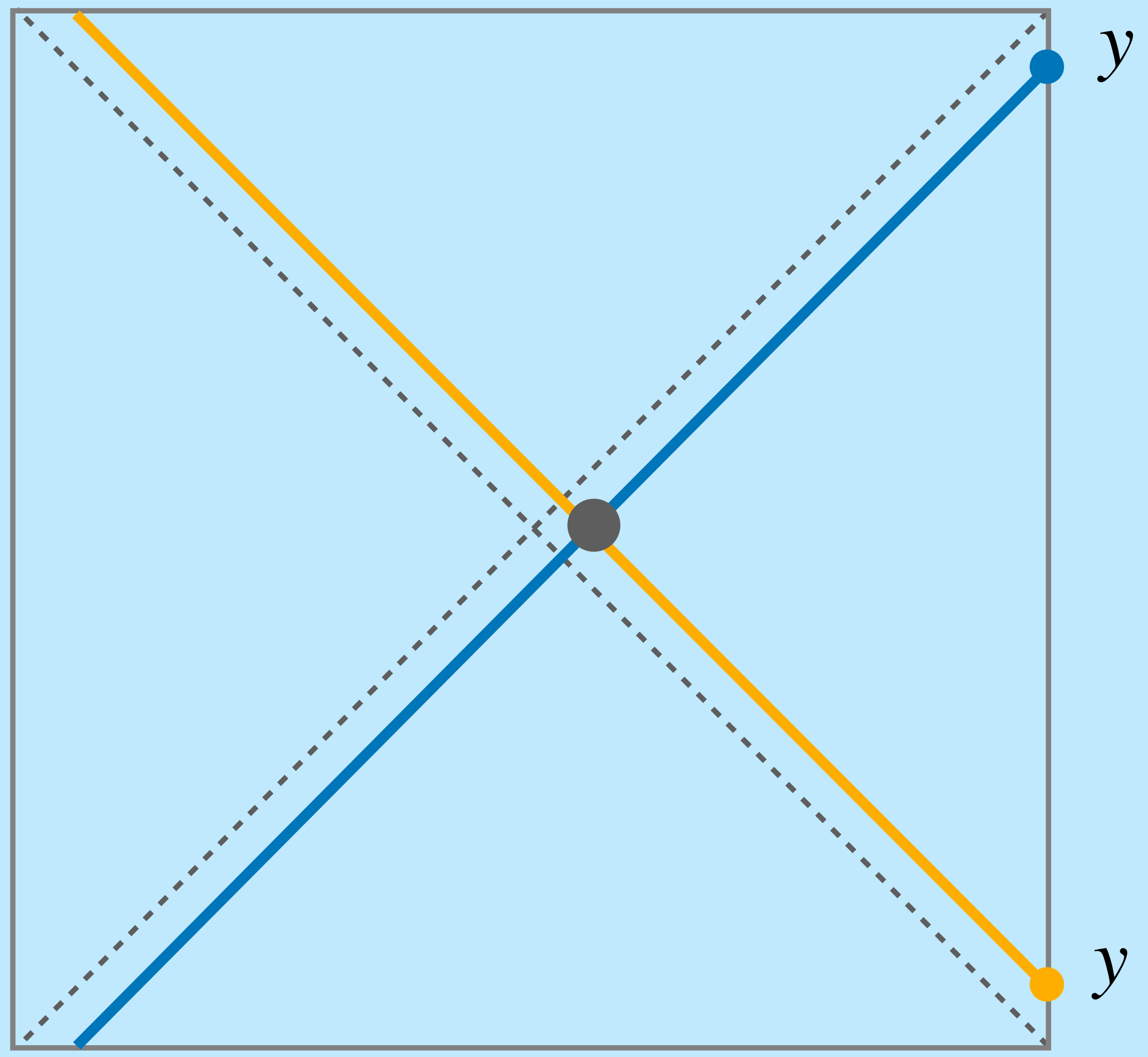

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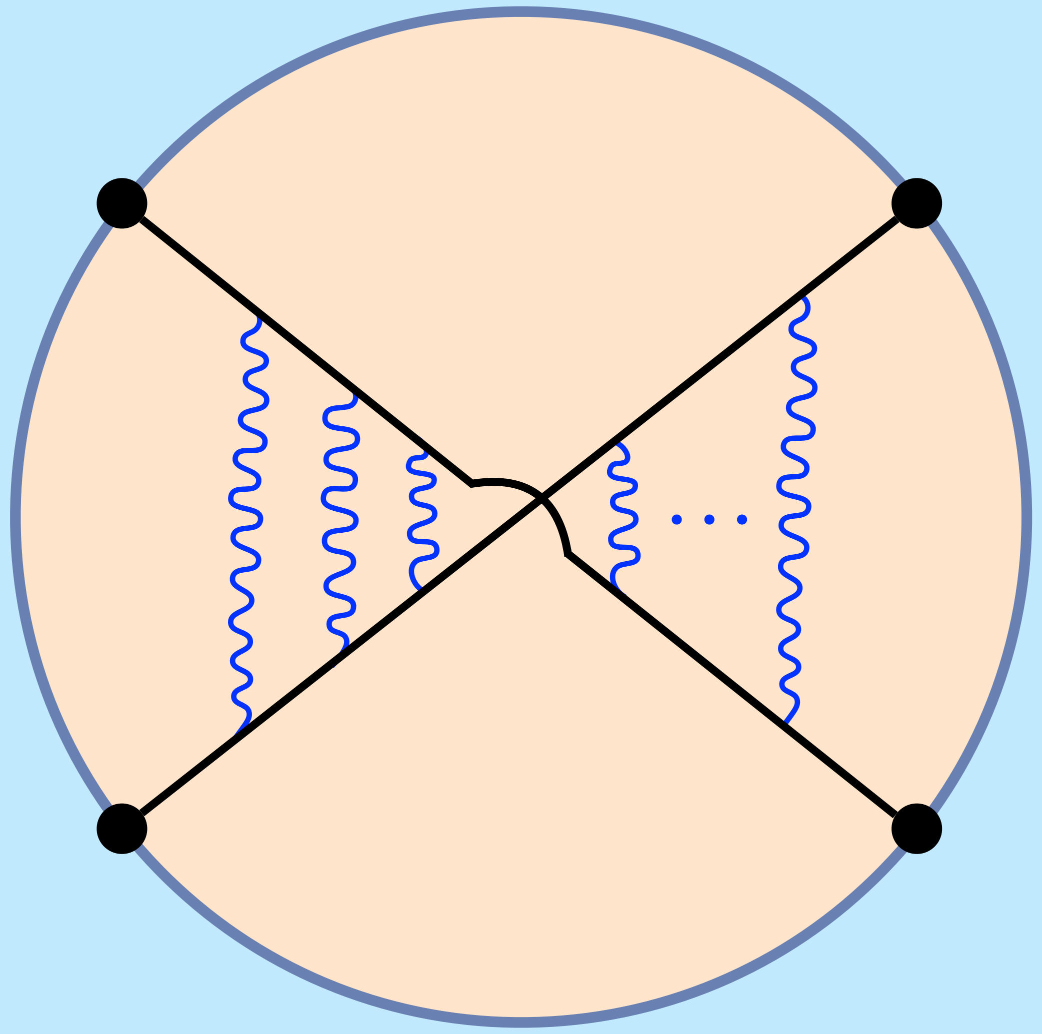

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$$\langle y_1 y_3 y_2 y_4 \rangle = \int \mathscr{D}\alpha e^{-S_{L,2}[\alpha]} B(x_1, x_2) B(x_3, x_4)$$

$$\xrightarrow{t\to\infty} \frac{1}{\kappa^2} U(2,1,\kappa^{-1})$$

Thank you for listening