Surface defects, phase transitions and supersymmetric black holes

Mengyang Zhang
(Based on 2306.05463 with Yiming Chen, Matthew Heydeman, Yifan Wang)

Princeton University
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Motivation and overview

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- The superconformal index could be modified by inserting the extended observable. The extended observable should be at least $\frac{1}{16}$-BPS and extend along temporal direction in order to probe the black hole geometry. The modified index reflects the entropy of (BH+string/brane) system.

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- This 4d-2d system has a holographic dual: Single surface defect insertion $\longleftrightarrow$ Single probe D3 brane extending into the bulk and intersecting with the stack of N D3 branes (Constable, Erdmenger, Guralnik, Kirsch)

	0	1	2	3	4	5	6	7	8	9
N D3-branes	$\times$	$\times$	$\times$	$\times$						
Probe D3-brane	$\times$	$\times$			$\times$	$\times$				

Table: Bulk branes configuration

Superconformal index with surface defects

- The Hilbert space over the spatial S^3 is modified by including local operators on the defect: $\mathcal{H}(S^3) \to \mathcal{H}_{\mathcal{D}}(S^3)$. The defect index can be defined as a supersymmetric partition function tracing over the defect Hilbert space $\mathcal{H}_{\mathcal{D}}(S^3)$:

$$\mathcal{I}_{\mathcal{D}} = \mathrm{Tr}_{\mathcal{H}_{\mathcal{D}}(\mathrm{S}^3)} \left((-1)^F e^{-\beta\{\mathcal{Q},\mathcal{Q}^\dagger\}} p^{J_1+\frac{1}{2}R_3} q^{J_2+\frac{1}{2}R_3} y_1^{\frac{1}{2}(R_1-R_3)} y_2^{\frac{1}{2}(R_2-R_3)} \right) .$$

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- The index calculation can be reduced to a unitary matrix integral (Gadde, Gukov; Nakayama; Drukker, Gomis, Matsuura):

$$\mathcal{I}_{\mathcal{D}}(p,q,y_i) = \int_{SU(N)} [dU]\, \mathcal{I}_{4d}(p,q,y_i;U)\mathcal{I}_{2d}(p,q,y_i;U)\,,$$

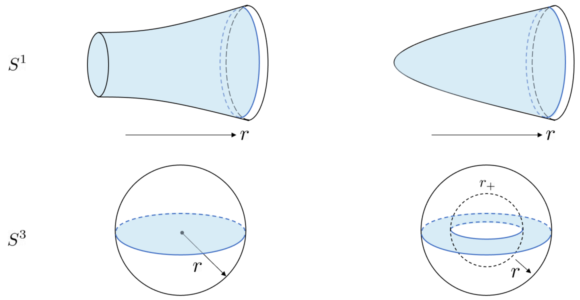
$\mathcal{I}_{\mathrm{4d}}$ and $\mathcal{I}_{\mathrm{2d}}$ are minimally coupled via SU(N) gauge fields.

Large N saddle-point approximation (U_* is the saddle w.r.t. $\mathcal{I}_{\mathrm{4d}}$):

$$\mathcal{I}_{\mathcal{D}}(p,q,y_i) = \sum_{\text{saddles } U_*} \mathcal{I}_{4d}(p,q,y_i;U_*)\mathcal{I}_{2d}(p,q,y_i;U_*) + \dots\,.$$

Dual D3 brane action

- Probe D3 brane wraps the AdS_2 cigar geometry in BH phase.



(a) Thermal AdS (b) Black hole

- Evaluate the usual DBI action

$$I_{\mathrm{D3},E} = -T_{\mathrm{D3}} \int \left(d^4x \sqrt{-\det(h_{\mathrm{D3}})} - P[C_{(4)}] \right) , \quad T_{\mathrm{D3}} = \mathcal{O}(N)$$

$$\langle \mathcal{D} \rangle = \mathcal{I}_{2d}(p, q, y_i; U_*) \sim e^{-I_{\mathrm{D3,E}}} .$$

Summary and Future direction

Conclusion

- Found Gukov-Witten surface defect as a supersymmetric order parameter of deconfinement phase transition: $\langle \mathcal{D} \rangle_{\text{thermal-AdS}} = e^{\mathcal{O}(1)}$, $\langle \mathcal{D} \rangle_{\text{BH}} = \exp\left\{ 2\pi i N \frac{(\sigma+\tau-1)^2}{9\sigma} \right\}$
- The holographic dual D3 brane, which interacts with the black hole, probes finer features of black hole physics

Future directions

- Other supersymmetric extended observables
- Half-BPS surface defects in other holographic theories
- Multiple surface defects insertion and backreaction to the geometry

Thank you!