Brane Fusion Frenzy

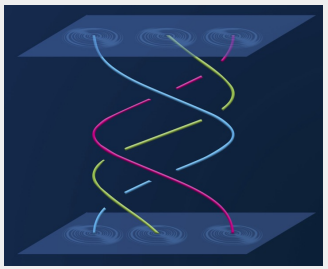
Non-Invertible Defect Fusion and Tachyon Condensation

Thomas Waddleton

in collaboration with Enoch Leung and Ibrahima Bah, arXiv:2306.15783

Johns Hopkins University

Simons Collaboration on Global Categorical Symmetries



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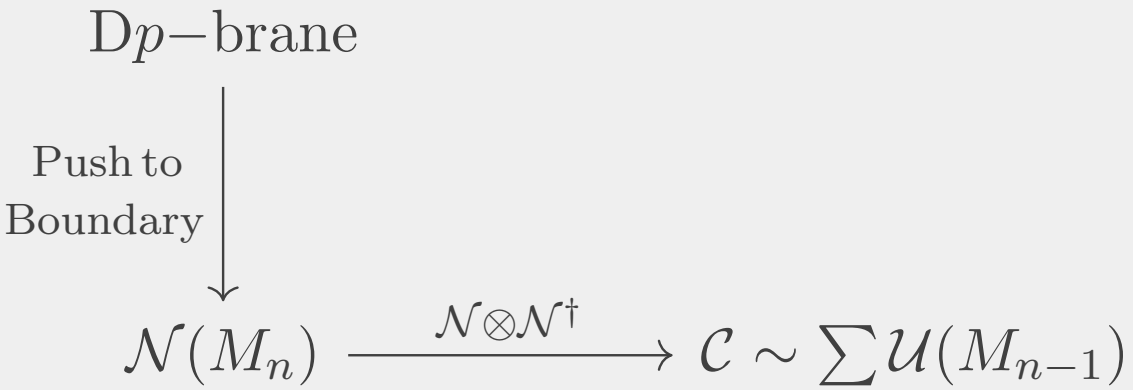
- Fusion coefficients given by **decoupled TFTs**, not numbers
  - $\mathcal{N}_a \otimes \mathcal{N}_b \simeq \mathcal{A} \otimes \mathcal{N}_{a+b}$

Completing the Square



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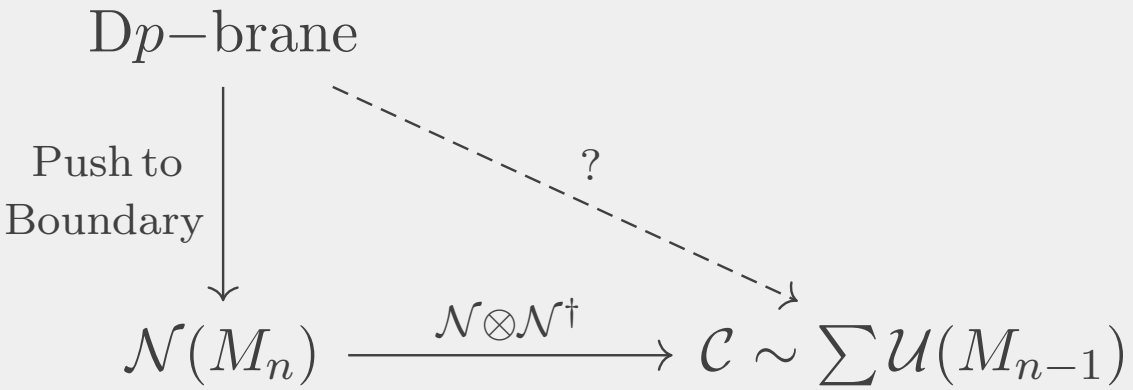

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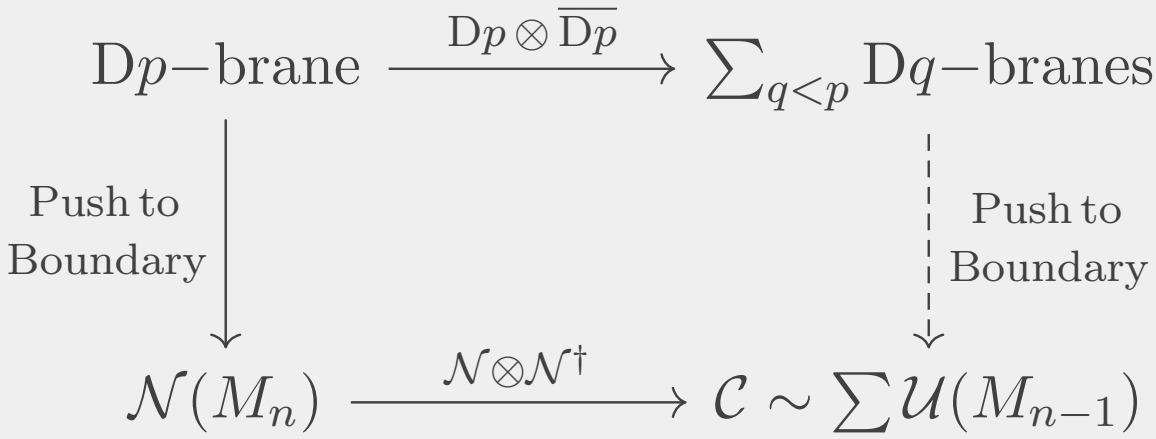

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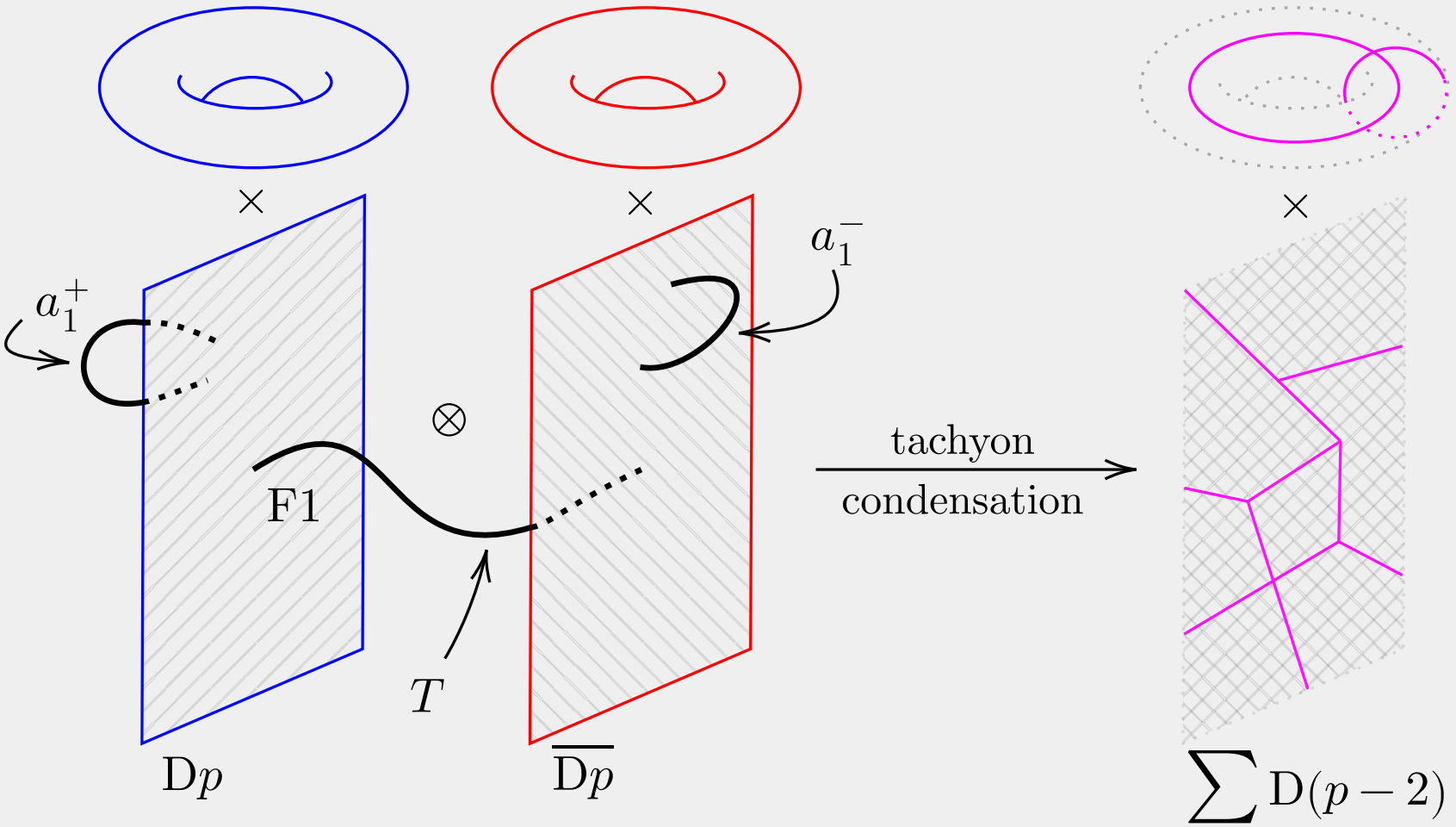
- Can we understand $\mathcal{C}$ directly from something stringy?
- Model with brane dynamics: get lower brane charges from **tachyon condensation** [Sen], [Witten]


a_1^+
a_1^-
F1
T
Dp
$\overline{\text{D}p}$
tachyon
condensation
$\sum \text{D}(p-2)$

What'd We Do?

- $\mathcal{W}^5 \times T^{1,1}$ with N D3-branes and M fractional D3-branes; dual to 4d $\mathcal{N} = 1$ $\mathfrak{su}(M)$ SYM after duality cascade

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- **Purely from branes**, see $\mathcal{N} \otimes \mathcal{N}^\dagger \sim \sum \mathcal{U}$!

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- Beginning to understand the bulk structure of all symmetry operators
  - Should be (higher) fusion categories
    [Copetti, Del Zotto, Ohmori, Wang], [Bhardwaj, Bottini, Schafer-Nameki, Tiwari]

Thank You!