Hearts of Darkness:

Nonsingular Black Holes Beyond General Relativity

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Puzzles in the Quantum Gravity Landscape

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Image credit: EHT collaboration

Mainly in collaboration with Carballo-Rubio, Liberati, Pacilio, Visser.

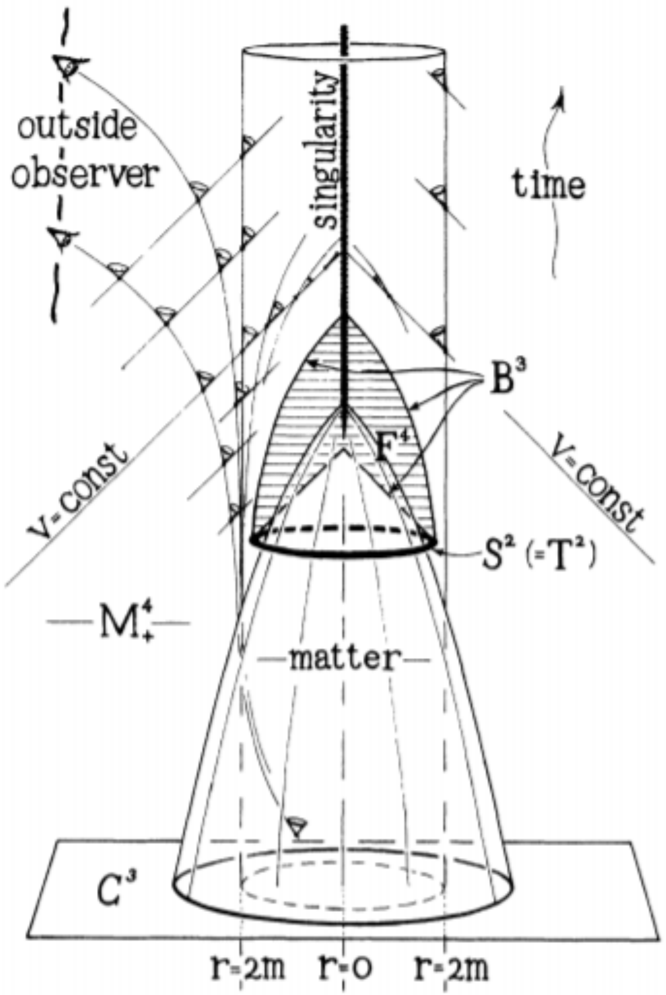
Introduction

General Relativity is an extremely elegant and successful theory.

Observations coming from the LIGO/Virgo, the EHT, and the GRAVITY collaborations are in agreement with the prediction of general relativity.

However, there are also reasons to extend GR. In particular, the theory predicts its own breakdown due to the formation of singularities

Weak Cosmic Censorship Conjecture:
Singularities are hidden from an observer at infinity by the event horizon.



It is tempting to assume that any effect of the singularity or of the regularisation will be hidden from us.

Why do we care: A lesson from Newtonian gravity

Let us imagine having no access to high energy particle or strong gravity phenomena.

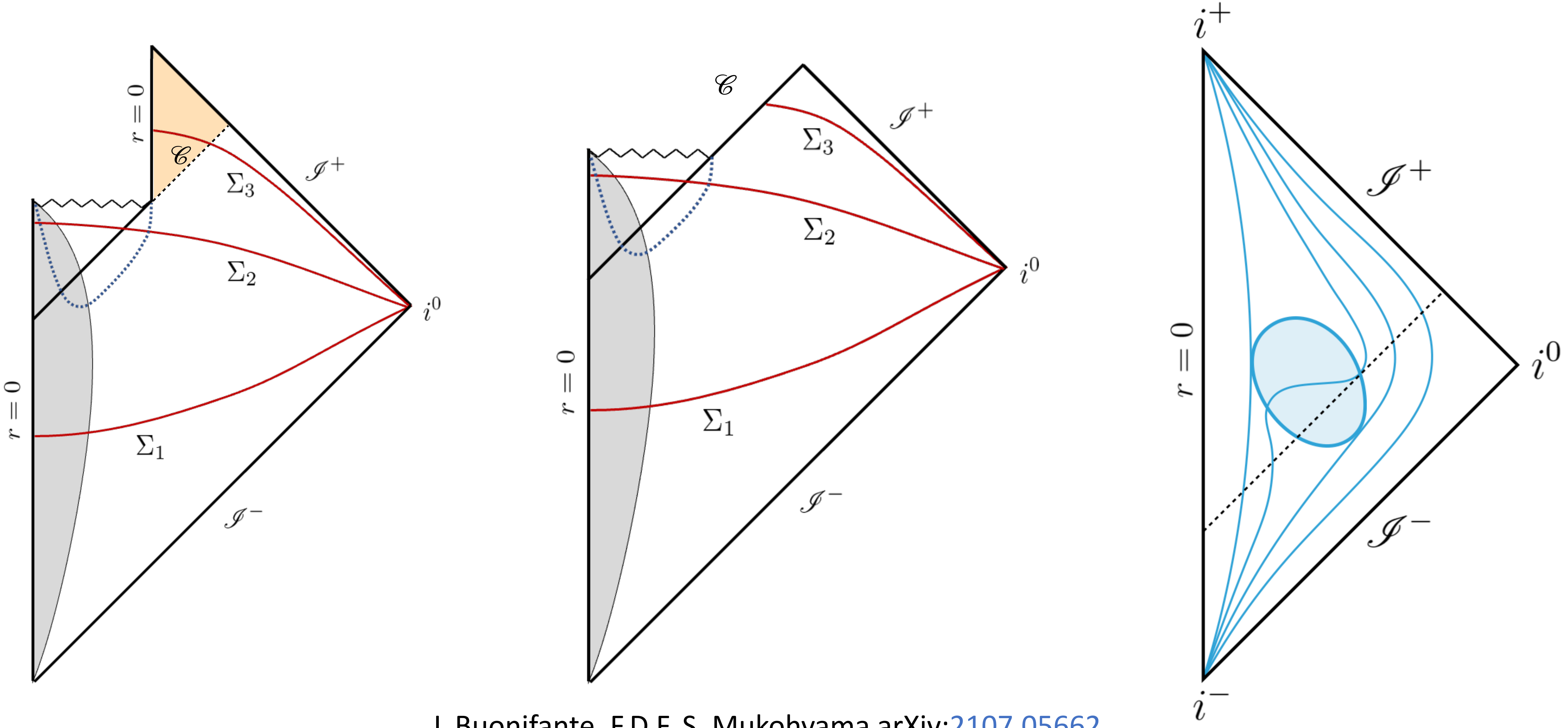
Newtonian gravity represent an excellent approximation.

Should we accept the impossibility of probing physics beyond Newton?

No, e.g. gravitational waves!

We should not exclude that modifications to general relativity can propagate outside black holes

Why do we care 2: Black hole evaporation



L.Buonifante, F.D.F, S. Mukohyama arXiv:2107.05662

Outline

Non-singular black holes candidates

Viability issues

Addressing the viability issues?

Non-singular black holes candidates

R. Carballo Rubio, F. Di Filippo, S. Liberati, M. Visser. *Opening the Pandora's box at the core of black holes.*
Class. Quant. Grav. 37 (2020) no.14, 145005, arXiv:1908.03261.

R. Carballo Rubio, F. Di Filippo, S. Liberati, M. Visser. *Geodesically complete black holes*.
Phys. Rev. D 101 (2020), 084047, arXiv:1911.11200.

R. Carballo Rubio, F. Di Filippo, S. Liberati, M. Visser. *Geodesically complete black holes in Lorentz-violating gravity*.
JHEP 02 (2022) 122, arXiv:2111.03113.

Penrose singularity (incompleteness) theorem

Consider spacetimes where a trapping surface is formed and

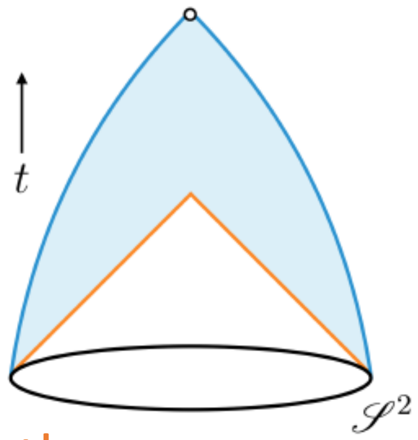
a) Pseudo-Riemannian geometry provides an adequate description of spacetime;
b) The spacetime is globally hyperbolic;
c) Null convergence condition $R_{ab}K^aK^b \geq 0$ holds;

Then

- The spacetime is geodesically incomplete.

We consider spacetimes where a trapping surface is formed and

a) Pseudo-Riemannian geometry provides an adequate description of spacetime;
b) The spacetime is global hyperbolic;
c) The spacetime is geodesically complete;
d) There are no curvature singularities.

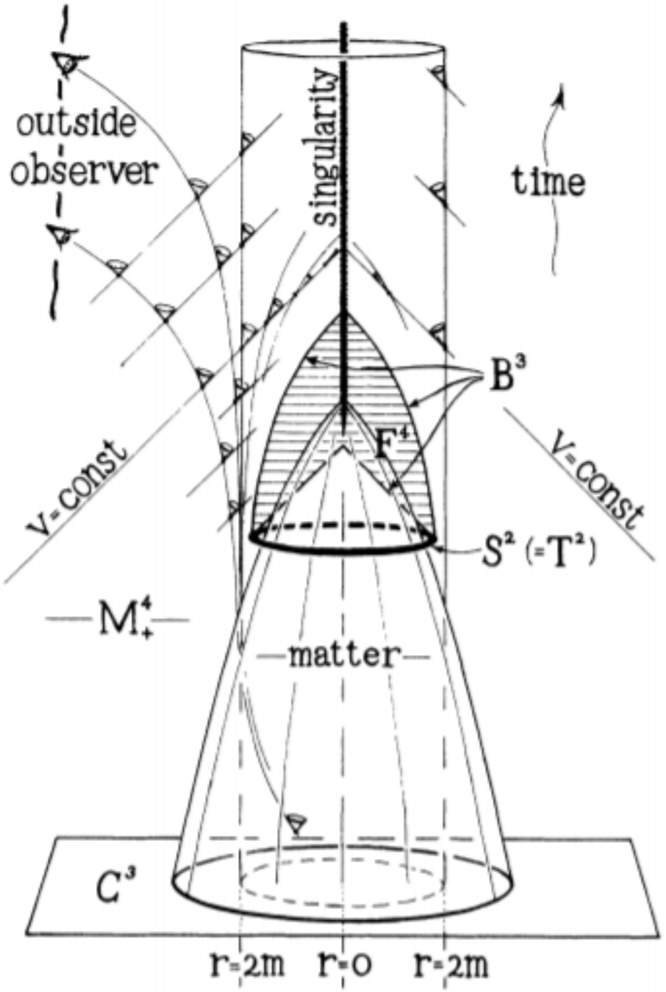


Classifying non-singular black holes corresponds to classify how to avoid the theorem

Singularity avoidance: possibilities

By studying the possible ways out of Penrose theorem we can classify non-singular geometries

- Regular black holes. Both outer and inner horizon;
- Wormholes. Local or global minimum radius surface, with or without outer horizon;
- Asymptotic regular black holes or wormholes. Inner horizon or minimum radius are pushed at infinite affine distance;
- Ultracompact horizonless objects. Surface close to the would be horizon



Viability and self-consistency

Regular black holes instability

R. Carballo Rubio, F. Di Filippo, S. Liberati, C. Pacilio, M. Visser. *On the viability of regular black holes.* JHEP 07 (2018), 023, arXiv:1805.02675.

R. Carballo Rubio, F. Di Filippo, S. Liberati, C. Pacilio, M. Visser. *Inner horizon instability and the unstable cores of regular black holes.* *JHEP* 05 (2021) 132. arXiv:2101.05006.

R. Carballo Rubio, F. Di Filippo, S. Liberati, C. Pacilio, M. Visser. *Regular black holes without mass inflation instability.* *JHEP* 09 (2022) 118. arXiv:2205.13556.

Regular black holes

Let us start studying a static configuration, the analysis will trivially extend to the dynamical case.

$$ds^2 = -\,e^{-2\phi(r)}F(r)dv^2 + 2e^{-\phi(r)}dvdr + r^2d\Omega^2$$

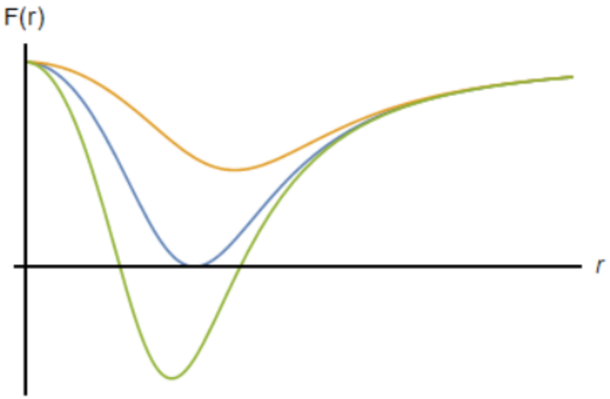
The horizon condition is $F(r) = 0$.

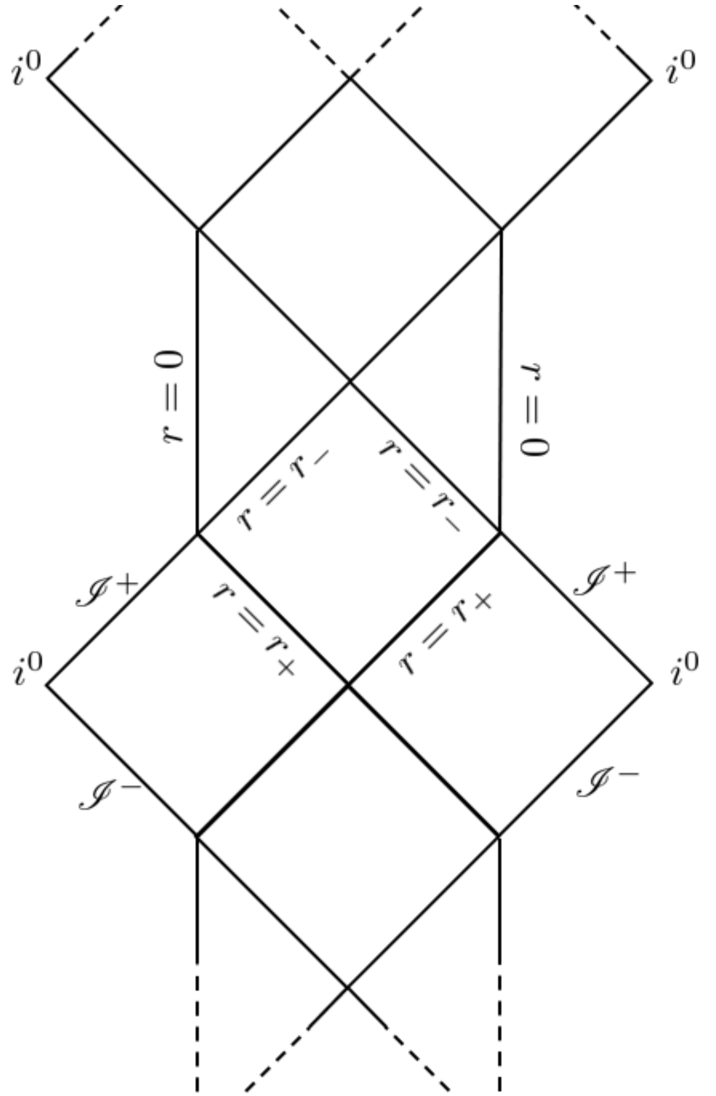
- $\lim_{r->0} F(r) = 1$
- $\lim_{r->\infty} F(r) = 1$

There is an even number of horizons

The surface gravity:

$$\kappa_{\pm} = \frac{1}{2}e^{-\phi(r_{\pm})}\frac{dF}{dr}\Big|_{r=r_{\pm}} \quad \Longrightarrow \quad \kappa_{-} < 0\,, \kappa_{+} > 0\,.$$





Examples of regular black holes

$$ds^2 = -e^{-2\phi(r)}F(r)dv^2 + 2e^{-\phi(r)}dvdr + r^2d\Omega^2 \qquad F(r) = 1 - \frac{2m(r)}{r}$$

Bardeen regular black hole

$$F = 1 - \frac{2Mr^2}{\left(r^2 + l^2\right)^{3/2}} \qquad \phi = 0$$

Hayward regular black hole

$$F = 1 - \frac{2Mr^2}{r^3 + 2Ml^2} \qquad \phi = 0$$

Dymnikova regular black hole

$$F = 1 - 2Mr^2\left(1 - e^{-r^3/2Ml^2}\right) \qquad \phi = 0$$

Reissner–Nordström Black Hole

The causal structure is equivalent to the one of a Reissner-Nördstrom black hole

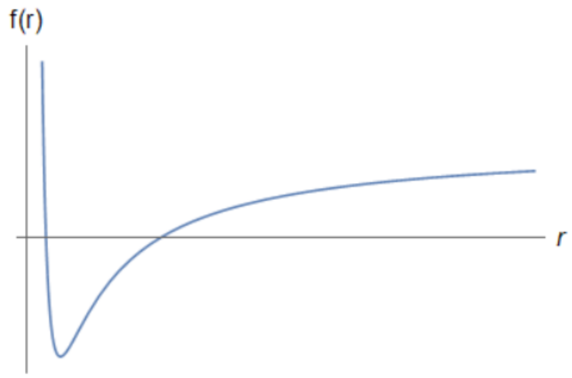
$$ds^2 = -\left(1 - \frac{2M}{r} + \frac{q^2}{r^2}\right)dt^2 + \left(1 - \frac{2M}{r} + \frac{q^2}{r^2}\right)^{-1} dr^2 + r^2 d\Omega^2 \,.$$

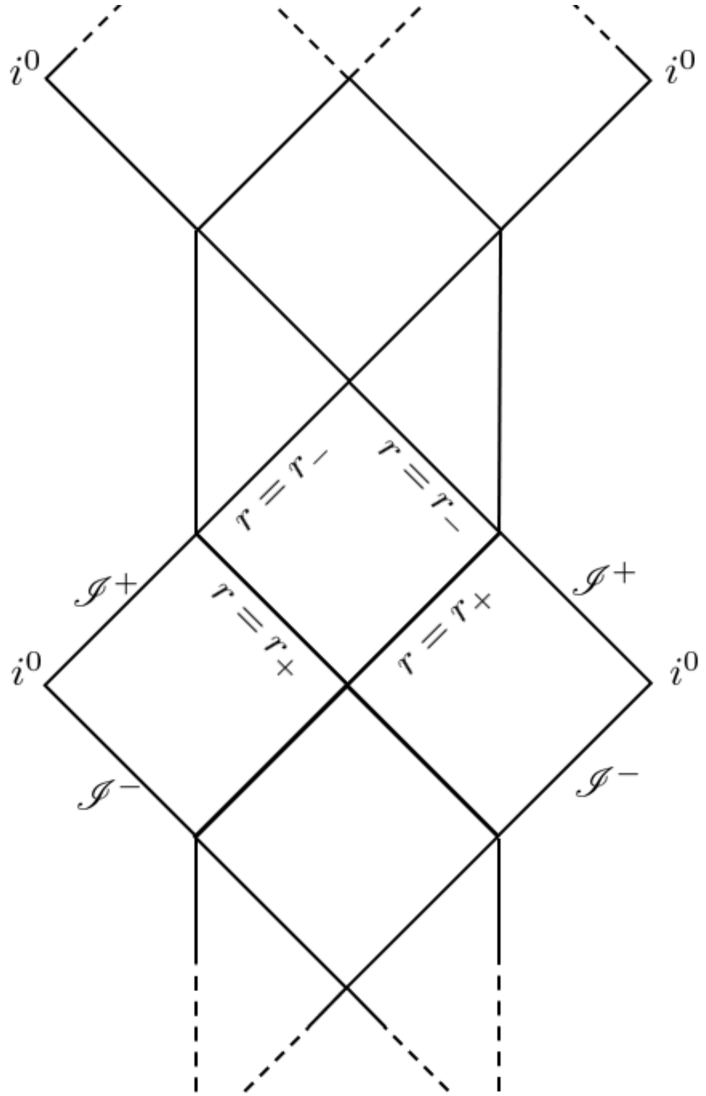
It is well known that the inner horizon is unstable [Poisson, Israel; Ori].

Small perturbations produce a null or spacelike singularity.

This is actually a welcome feature in this case as it saves the strong cosmic censorship conjecture

SCC: the classical fate of all observers should be predictable from the initial data





Mass Inflation

We want to study the stability of the inner horizon.

Problem:

We do not know the field equations.

Solution:

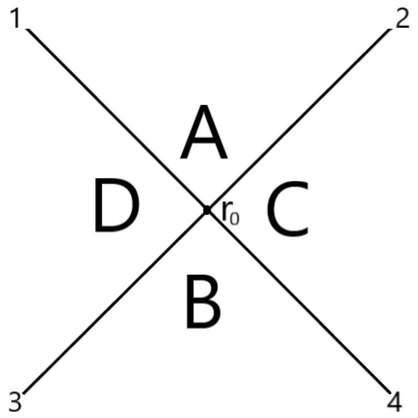
Consider a geometrical approach with few assumptions

What are these assumptions? What are the limitations of the approach?

Double null shell

Let us now add some perturbation to the regular black hole background

Consider an ingoing and an outgoing null shell colliding to form two new null shells;



Is this a reasonable type of perturbation? Yes, close to the inner horizon

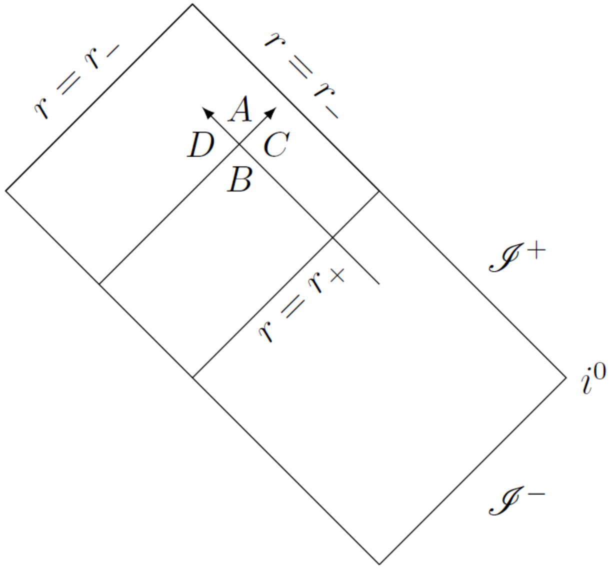
On purely geometrical ground

[C. Barrabes, W. Israel, E. Poisson Phys. Rev. D 41 (1990)]

$$g_A^{rr}(r_0)\,g_B^{rr}(r_0) = g_C^{rr}(r_0)\,g_D^{rr}(r_0)$$

With

$$g^{rr} = F(r) = 1 - \frac{2m(r)}{r}$$



It follows

$$m_A(r_0) = m_B(r_0) + m_{in}(r_0) + m_{out}(r_0) - \frac{2m_{in}(r_0)m_{out}(r_0)}{r_0 F_B(r_0)}$$

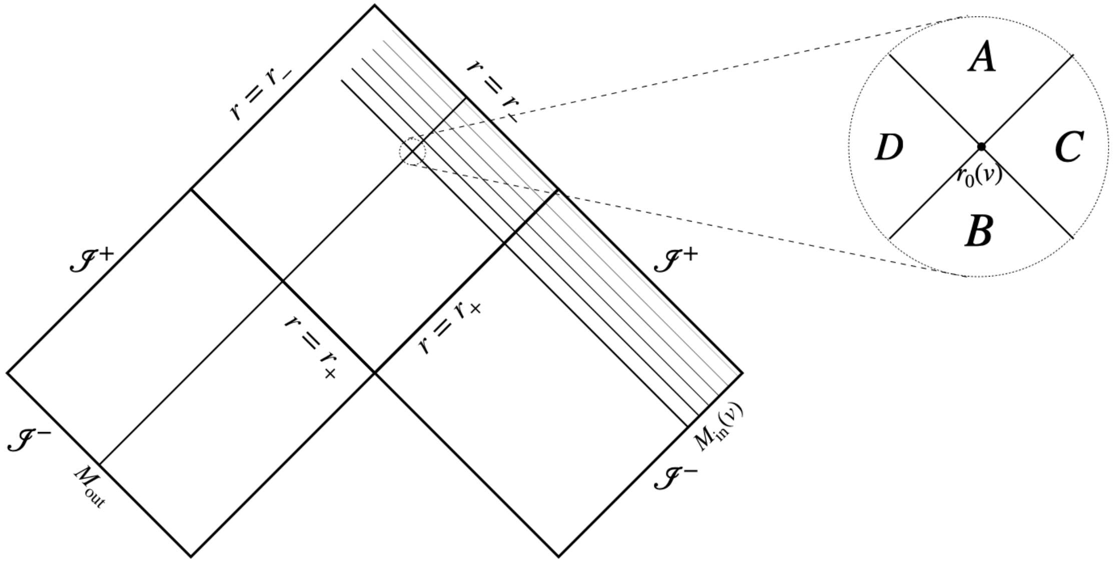
Where

$$m_{in}(r_0) := m_C(r_0) - m_B(r_0)\,, \quad m_{out}(r_0) := m_D(r_0) - m_B(r_0)\,.$$

Double null shell

What are m_{in} and m_{out} ? We assume that the energy of each shell shifts the asymptotic mass.

$$m_{out} \approx \left.\frac{\partial m}{\partial M}\right|_{r=r_-} M_{out}\,, \qquad m_{in} \approx \left.\frac{\partial m}{\partial M}\right|_{r=r_-} M_{in}(v)\,.$$



Price law:

[Price 1972; Gundlach, Price, Pullin 1994; Dafermos, Rodnianski 2005]

$$M_{in} \propto v^{-p}$$

Behavior of $F_B(r_0)$:

$$|F_B| \propto e^{-|\kappa_-|v}$$

Putting these two together

$$\boxed{m_A \propto v^{-p} e^{|\kappa_-|v}}$$

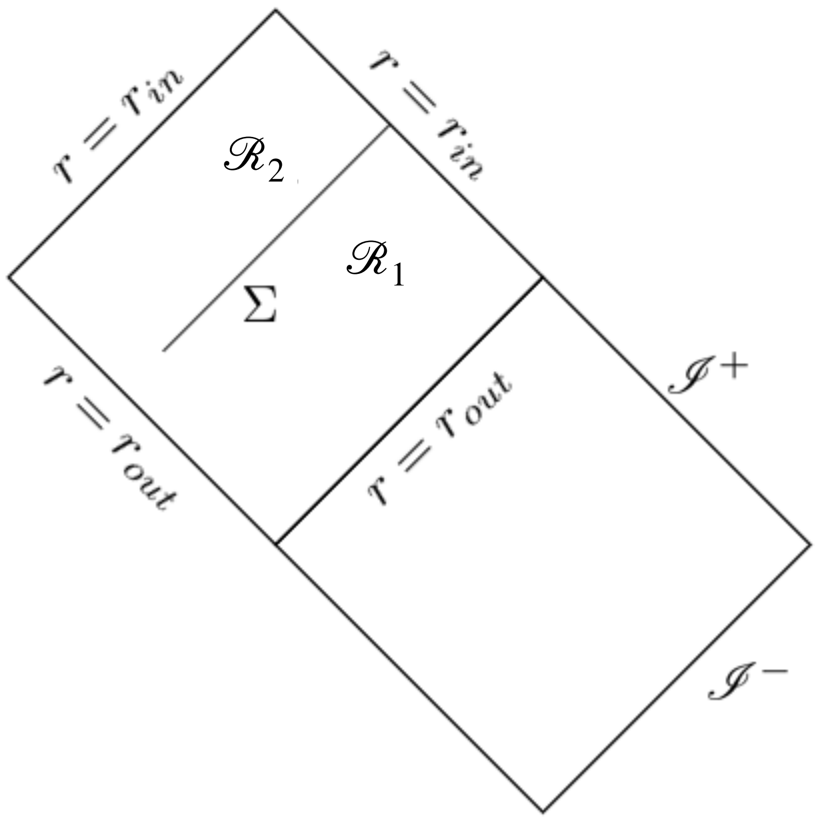
A small perturbation has a huge backreaction on the geometry.

Modified Ori model

The background geometry is replaced by a continuous flux of energy described by the Vaidya spacetime [Ori 1991].

$$ds^2 = -F(v,r)dv^2 + 2dv\,dr + r^2 d\Omega^2 \qquad F(v,r) = 1 - \frac{2m(v,r)}{r}$$

The shell divides the spacetime into two regions



For a pressureless shell, we can obtain

$$\frac{1}{f_2}\frac{\partial m_2}{\partial v} = \frac{1}{f_1}\frac{\partial m_1}{\partial v}\,.$$

The (late time) behavior of $M_1(v)$ is fixed by the Price law

$$M_1(v) = M_0 - \frac{\beta}{v^p}\,.$$

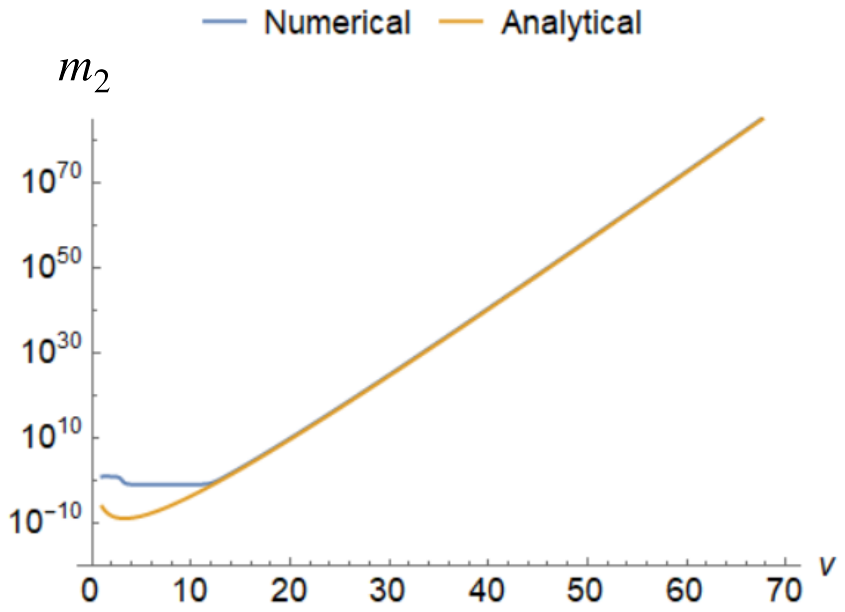
With this information we obtain

$$\frac{dm_2}{dv} = \left(|\kappa_-| - \frac{p+1}{v}\right)\left(m_2 - \frac{r_-}{2}\right) + \frac{dR}{dv}\frac{\partial M_2}{\partial r}$$

Numerical results

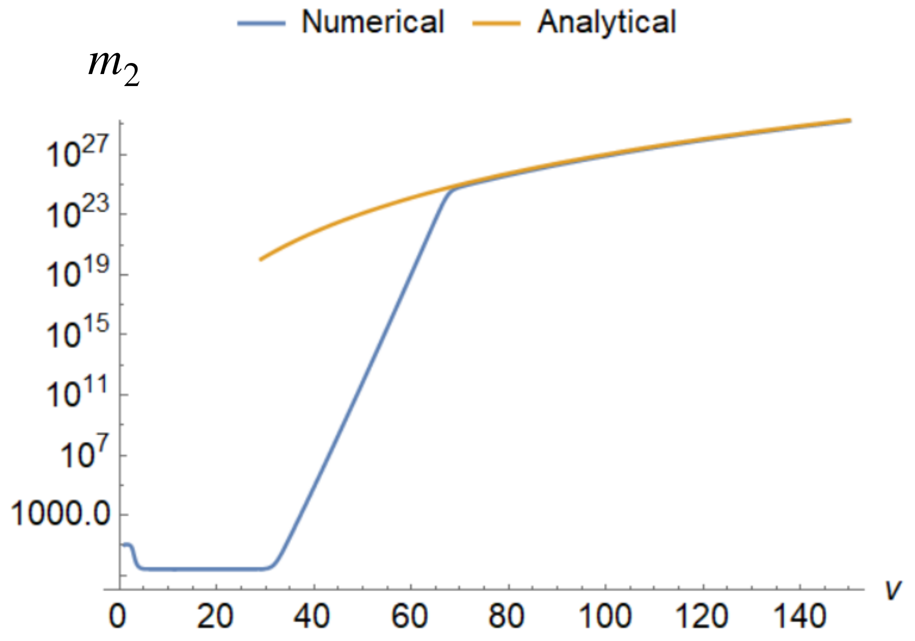
Bardeen:

$$F_{1,2} = 1 - \frac{2M_{1,2}r^2}{\left(r^2 + l^2\right)^{3/2}}$$



Hayward:

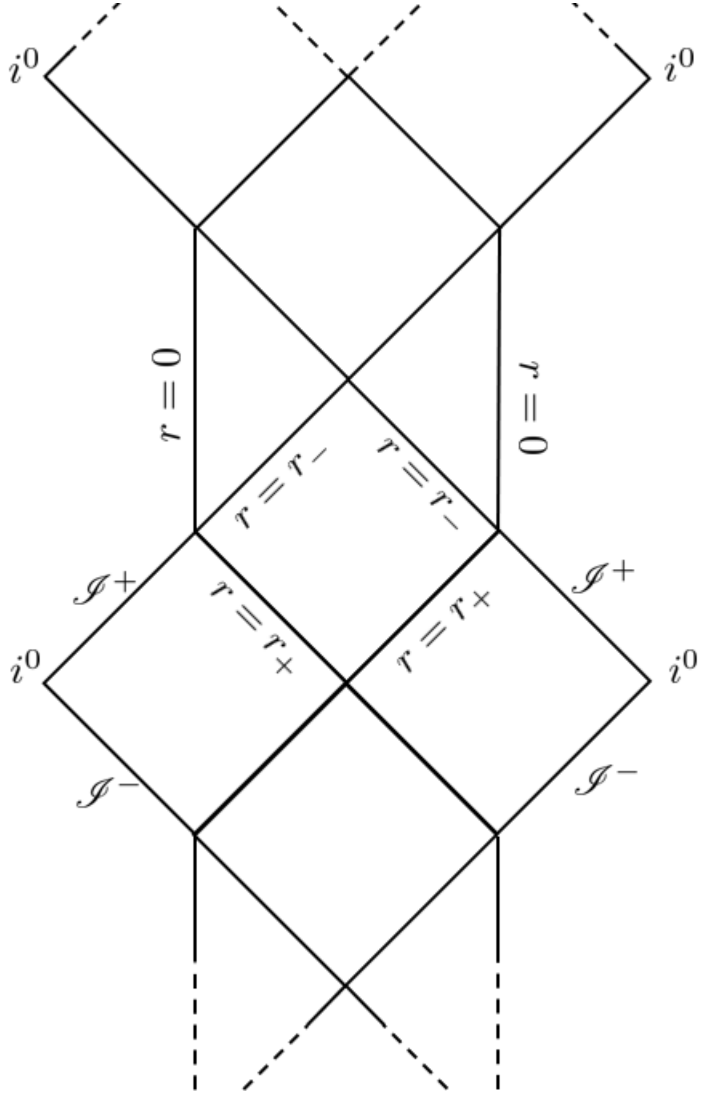
$$F_{1,2} = 1 - \frac{2M_{1,2}r^2}{r^3 + 2M_{1,2}l^2}$$



Parameters: $\beta = 1, \quad p = 12, \quad M_0 := M_1(v = 1) = 10, \quad l = 1, \quad R(v = 1) = 5, \quad M_2(v = 1) = M_0 + 1.$

A small perturbation has a huge backreaction on the geometry.

Dynamical regular black holes

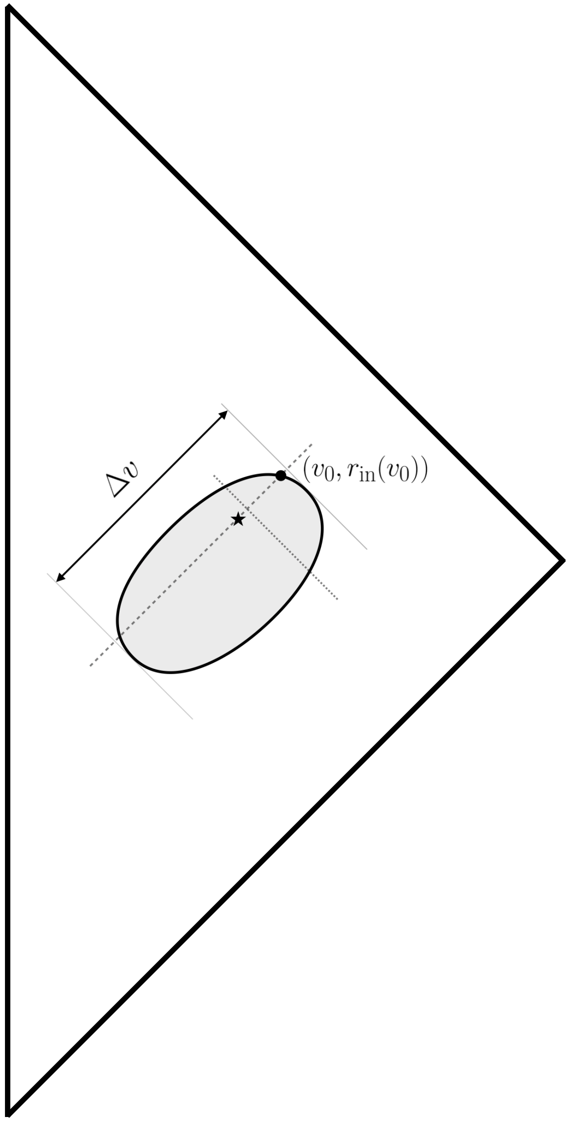


We only discussed the static case.

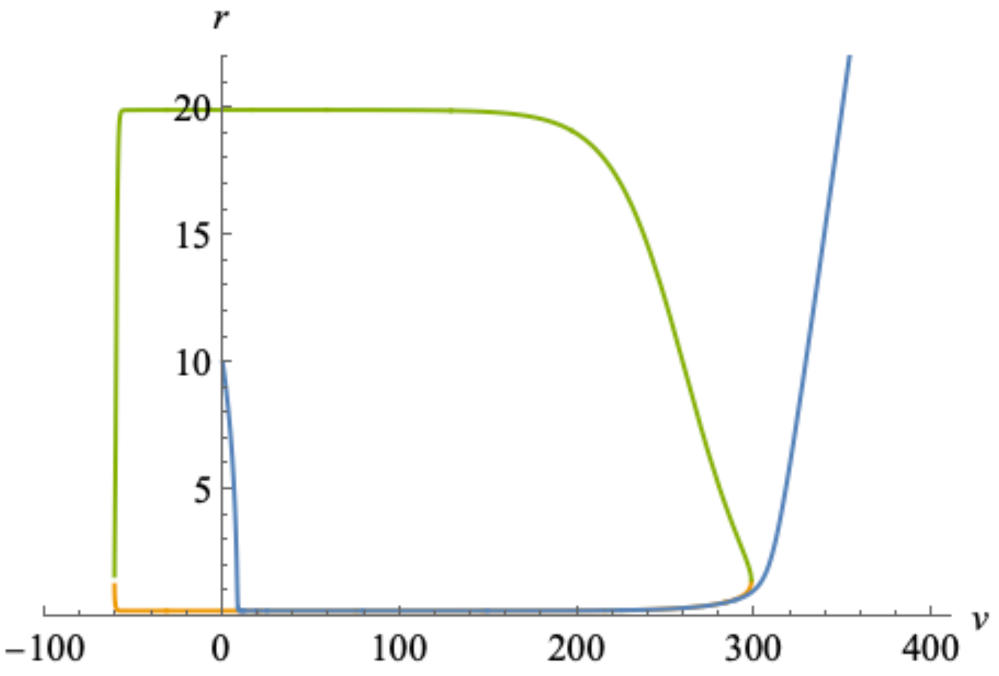
For the eternal geometry, the inner horizon is also a Cauchy horizon.

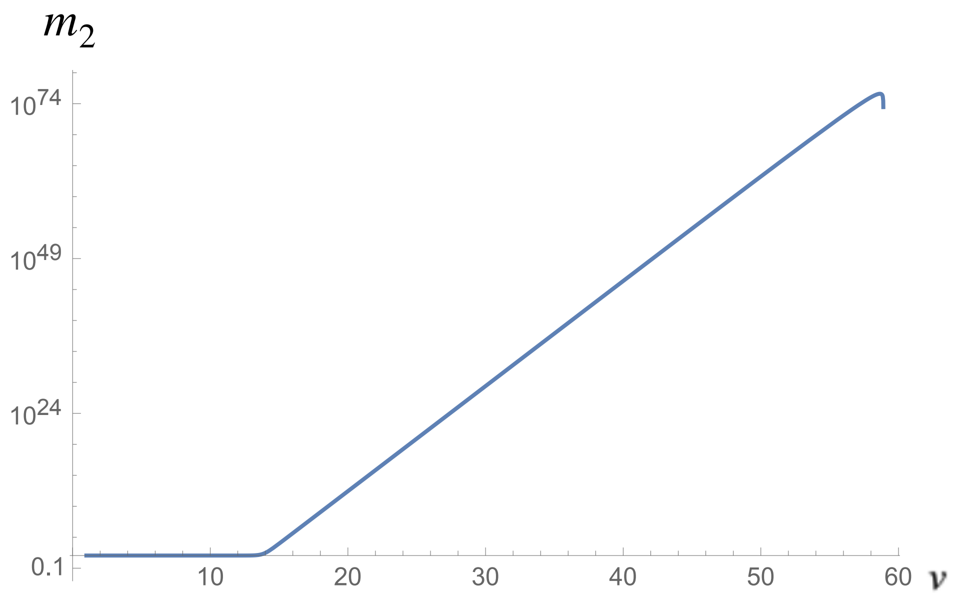
The causal structure of a dynamical regular black hole is very different.

However, we should reproduce the same result if there is the geometry varies on a timescale much longer than the instability timescale.



Dynamical regular black holes





The instability grows similarly to the eternal case until the shell escapes the trapped region

Work in preparation arXiv:23xx.xxxxx

Viability and self-consistency

Horizonless ultracompact objects instability

Inner light-ring instability

A horizonless space-time must have a second light ring [Cunha et al. 2017];

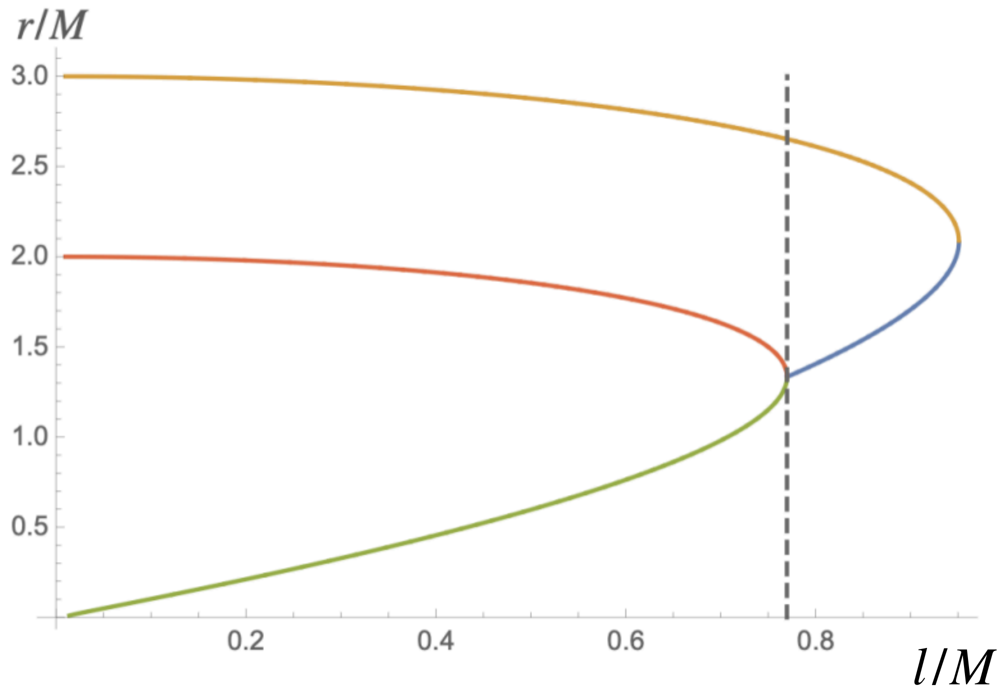
The inner light ring corresponds to a minimum of the effective potential;

For some boson stars models this leads to a non-linear instability [Cunha et al. 2022];

It seems that as soon as we get rid of the inner horizon instability, we get another source of instability.

Does this mean that there is no stable alternative to BH?

$$ds^2 = -\left(1 - \frac{2Mr^2}{r^3 + 2Ml^2}\right)dv^2 + 2dvdr + r^2 d\Omega^2$$



R. Carballo Rubio, F. D. F. , S. Liberati, M. Visser. arXiv:2211.05817

Addressing the viability issues?

R. Carballo Rubio, F. Di Filippo, S. Liberati, C. Pacilio, M. Visser. *Regular black holes without mass inflation instability.* *JHEP* 09 (2022) 118. arXiv:2205.13556.

Inner extremal regular black holes

The instability is driven by the surface gravity at the inner horizon. $M \propto e^{|\kappa_-| v}$

Is it possible to tame the instability by building an "inner extremal" regular black hole with $\kappa_- = 0$ and $\kappa_+ \neq 0$?

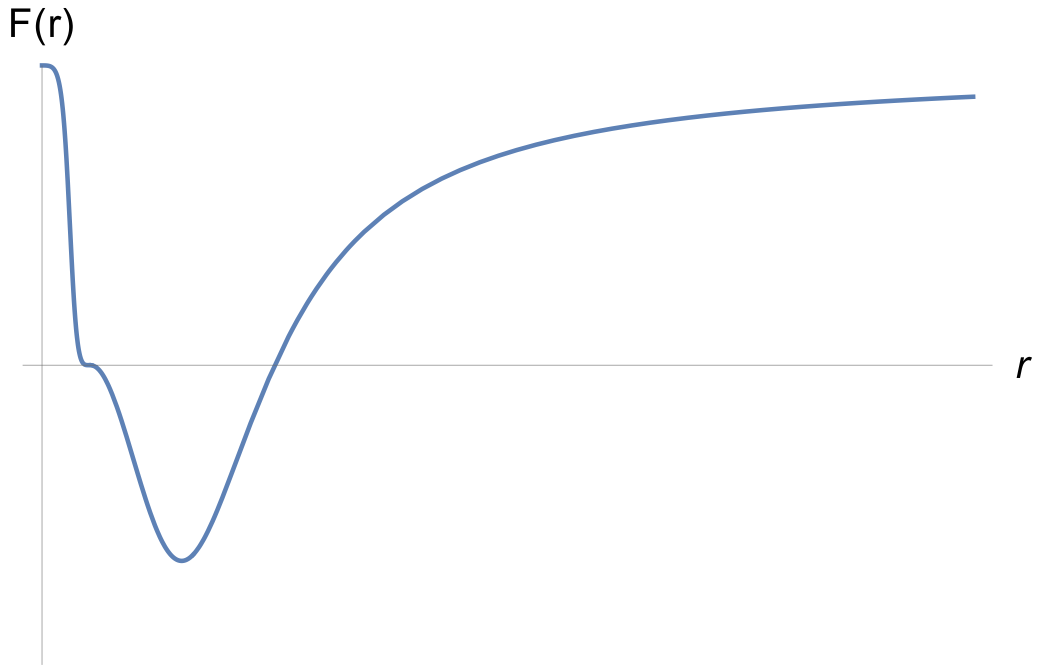
$$\kappa_\pm = \frac{1}{2} e^{-\phi(r_\pm)} \frac{dF}{dr}\bigg|_{r=r_\pm} .$$

It is easy to construct such geometry

$$ds^2 = -F(r)dv^2 + 2dvdr + r^2 d\Omega^2,$$

with

$$F = \frac{(r-r_-)^3(r-r_+)}{(r-r_-)^3(r-r_+) + 2Mr^3 + \left(a_2 - 3r_-\left(r_+ + r_-\right)\right)} .$$



Double null shells

We still obtain

$$m_A(r_0) = m_B(r_0) + m_{in}(r_0) + m_{out}(r_0) - \frac{2m_{in}(r_0)m_{out}(r_0)}{r_0 F_B(r_0)}$$

However,

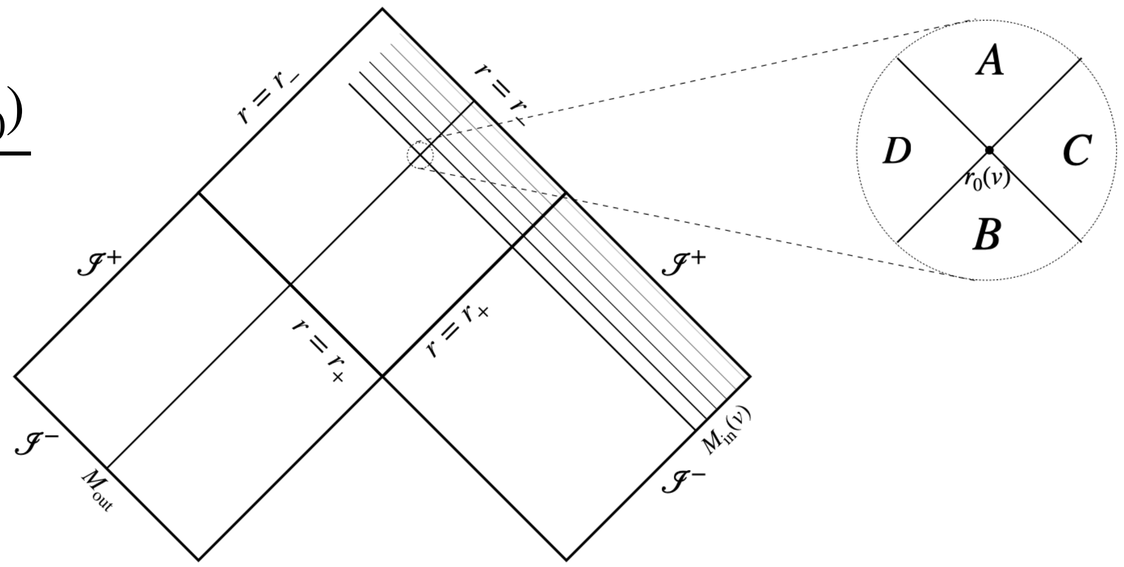
$$m_{in} \propto v^{-1} M_{in}(v) \qquad m_{out} \propto v^{-1} M_{out}\,.$$

And

$$F_B \propto v^{-3/2}.$$

Leading to

$$\boxed{m_A(r_0) = m_B(r_0) + h(M) M_{in} v^{-1} + h(M) M_{out} v^{-1} - \tilde{h}(M) v^{-1/2}\,.}$$

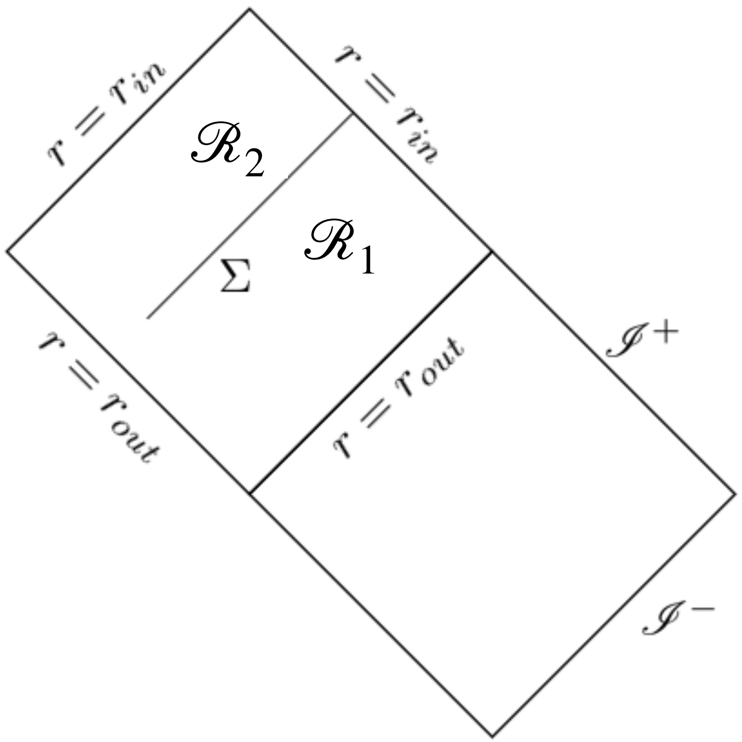


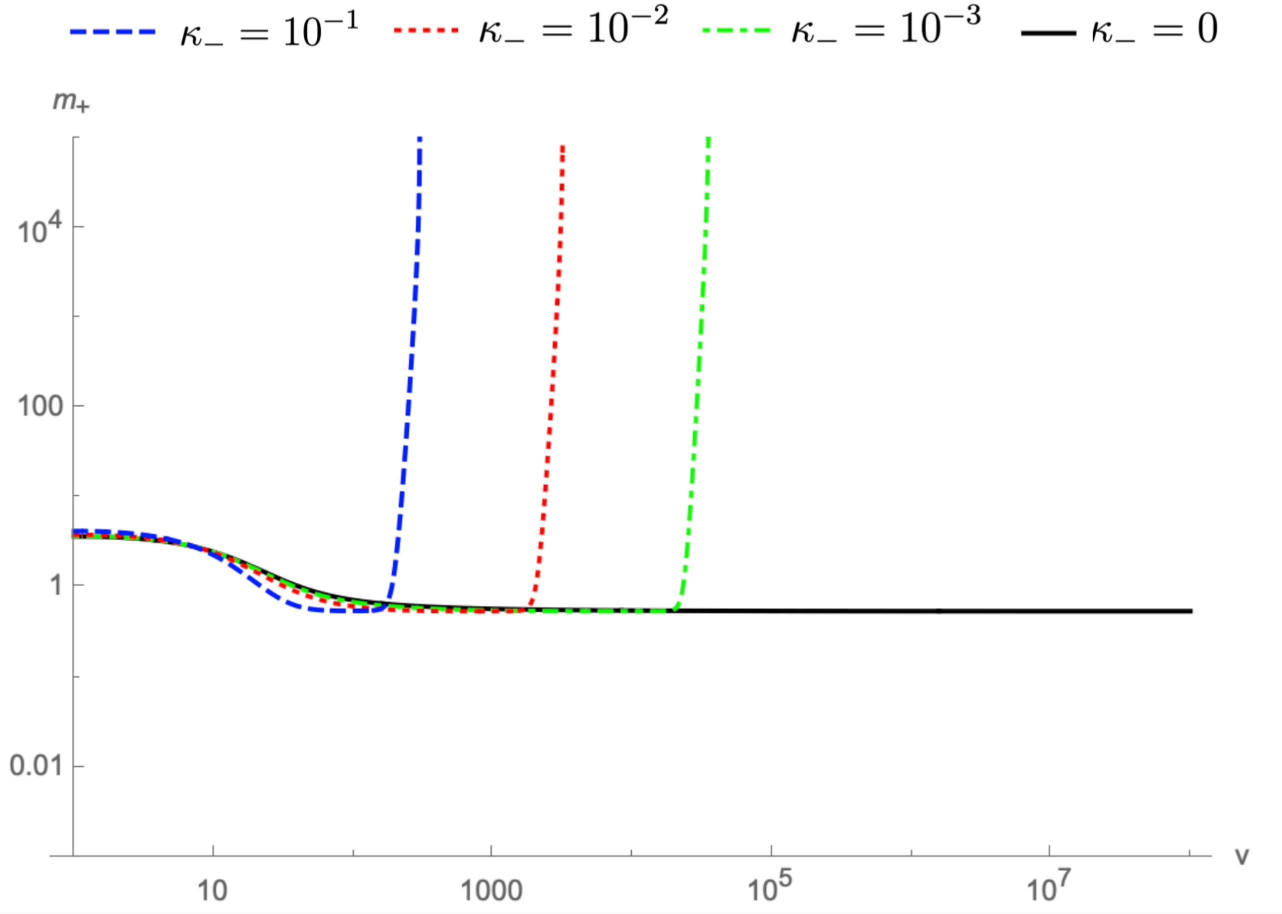
A small perturbation causes a small effect!

Modified Ori model

We can also repeat the numerical analysis for the Ori model.

$$F = \frac{(r-r_-)(r-kr_-)^2(r-r_+)}{(r-r_-)^3(r-r_+) + 2Mr^3 + \left(a_2 - 3r_-\left(r_+ + r_-\right)\right)}$$





No traces of instability!

Are inner extremal black holes reasonable or fine tuned?

Quick answer:

I don’t know!

Are inner extremal black holes reasonable or fine tuned?

It is interesting to note the existence of a (classically) stable regular black hole geometry.

At this stage, there is no motivation to consider this metric.

Regular black holes arise in several quantum gravity-inspired toy models, always with $\kappa_- \neq 0$.

Can we make sense of an inner extremal regular black hole?

What is the backreaction of the mass inflation instability?

- In GR, this usually leads to a singularity.
- Alternatively:
  - The inner horizon expands (might be good for observations);
  - The value of the surface gravity decreases (might provide a conservative resolution to the singularity problem).

Inner light ring instability

The presence of an inner light ring is universal.

The mechanism linking the stable light ring to the piling up of perturbation is also universal.

Is the instability universal? Can we build a stable ultra-compact object?

Boson stars solutions only exist up to a maximum mass.

Energy piling up at the minimum of the effective potential must lead to an instability.

What about objects supported by semiclassical effects?

$$R_{ab} - \frac{1}{2} g_{ab} R = \frac{8\pi G}{c^4} < \hat{T}_{ab} >$$

Final remarks

- Black holes hide theoretical evidence of the failure of general relativity.

- It is possible to classify any non-singular black hole spacetime with only a few classes.

- Regular black holes and ultracompact objects seem to be unstable.

- However, these geometries are generically unstable under small perturbations.

- Stable regular black hole geometries exist. Can we form them?

- Alternatively, does mass inflation lead to deviations at scales larger than Planck scale?

- Can we construct a stable ultracompact object?

- Yet a lot to discover about the dynamical formation mechanism.

Still work to do to obtain a viable alternative to black holes (at least for the classes discussed in this talk).
This might not be a negative feature!

Thank you!