The simplicial Lorentzian path integral and spin foams

Bianca Dittrich,
Perimeter Institute

Puzzles in the Quantum Gravity Landscape, Oct 2023
Perimeter Institute

Outlook

- (Effective) Spin foams
- Light cone structure in Lorentzian simplicial path integrals (for cosmology)
- Effective Spin foams for cosmology

Spin foams

Loop quantum gravity:

Rigorous background independent notion of quantum geometry. Closely related to TQFTs with defects.

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Areas have a discrete spectrum.

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Ryu-Takayanagi, tensor networks, bh entropy, …

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Path integral based on LQG quantum geometry.

Based on Plebanski formulation of gravity.

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Can spin foams lead to a gravitational dynamics?

[Bonzom, Hellmann, Kamiński, Han, …,Engle, Kamiński, Oliveira, Dona, Gozzini, …]

Classical Plebanski formulation requires constraining configuration to length metric space.

"Flatness problem" as a result of γ-anomaly.

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Is there an action for area metrics that leads to General Relativity in the low energy limit?

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Transparent encoding of the dynamics. Numerical simulations are faster by several magnitudes: seconds vs months.

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Perturbative continuum limit on lattice: $\lambda >> a$ [BD 2021, BD, Kogios 2022]

Results:

- Except for effective length metric all dof are Planck massive
- After integrating out all these additional dof:
- Leading order: (Linearized) Einstein-Hilbert action
- Next order: Weyl squared which comes from integrating out effective area metric
- Universality: Does not depend on details of spin foam models or on value of γ.

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- Derivation of action for area metrics from (modified) Plebanski action.
- Integrating out additional area metric dof's (linearized):

$$L_{eff} = L_{EH} - \frac{1}{4}\frac{1}{\Box - M(\gamma)^2}\text{Weyl}^2$$

Ghost-free!

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3 coupling constants G, Λ, γ: as needed in Asymptotic Safety, CDT, EDT. γ is an anisotropy parameter as in CDT.

Spin foams can lead to a gravitational dynamics in the continuum limit.

Rest of the talk: application to cosmology by symmetry reduction
with focus on Lorentzian features.

Our universe is Lorentzian … most of the times.

Spin foams are proper quantum mechanical path integrals.

$$Z \sim \int \mathscr{D}\mathrm{geom}\, \exp(iS(\mathrm{geom}))$$

For Euclidean geometries.

For Lorentzian geometries.

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Lorentzian path integral

Light cone structure

Consider Regge calculus.

How to compute it?

In Regge calculus. (Integral)

In Spin Foams. (Sum)

Light cone structure in simplicial path integrals

Lorentzian simplicial geometries: Regge calculus

[Regge 1961
Sorkin 1975, 2019
Asante, BD, Padua-Arguelles 2021]

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- Each simplex has a well defined Lorentzian geometry: it is Minkowski-flat.

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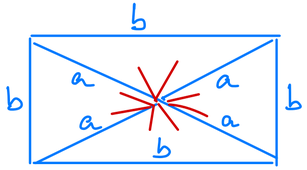
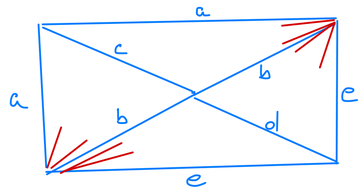
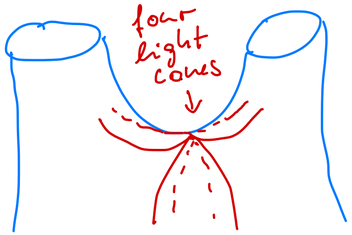
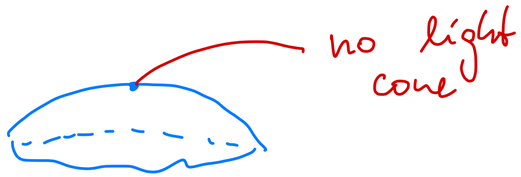
[Jordan, Loll 2013]









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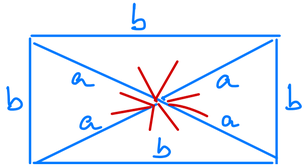

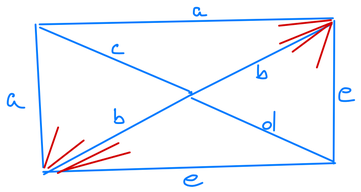

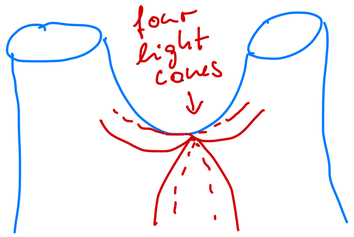

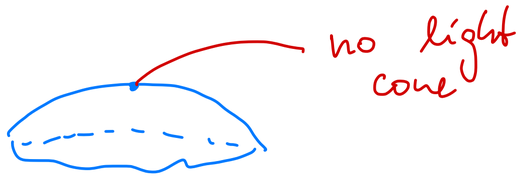

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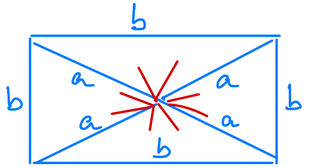

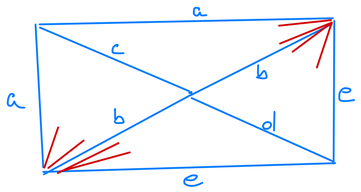

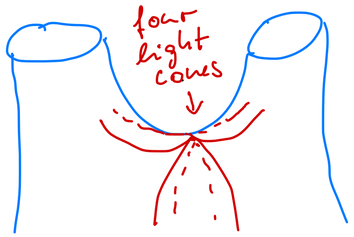

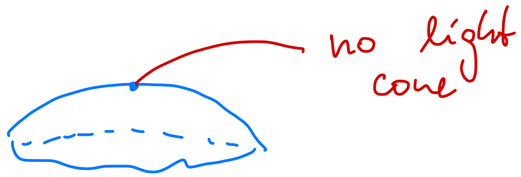

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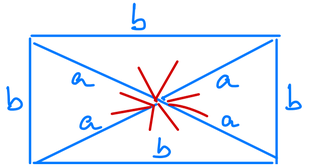

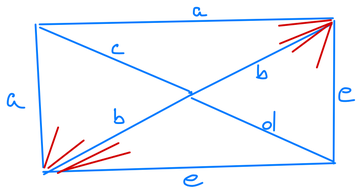

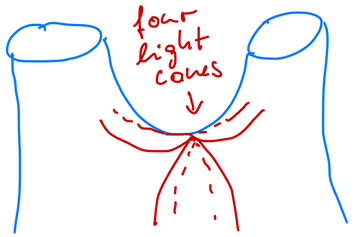

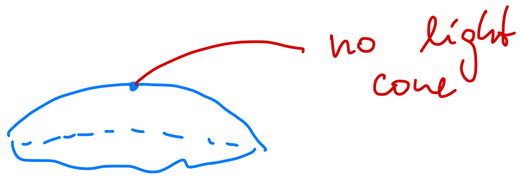

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?? Who ordered these configurations ??

?? What do we do with these configurations ??

Are these just annoying discretization artifacts?

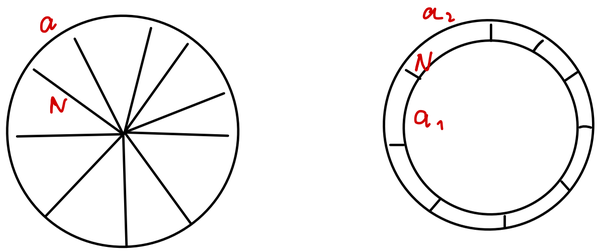
No. They can actually be useful.

- Imaginary metrics / imaginary terms in the action also appear in the continuum, e.g. for topology change. [Louko, Sorkin 1995, …., Witten 2022, …]
- Light cone irregularities (co-dimension 2 conical singularities) introduced by hand in Lorentizan continuum path integral [Marolf 2022]

- This talk: Important to get entropy for de Sitter space [BD, Jacobson, Padua-Arguelles TO APPEAR]
- This talk: Branch cut choice makes a more subtle choice in the continuum more obvious

Discretized de Sitter

One time evolution step in de Sitter



[BD, Gielen, Schander]

With symmetry reduction:

one integration variable ~ lapse

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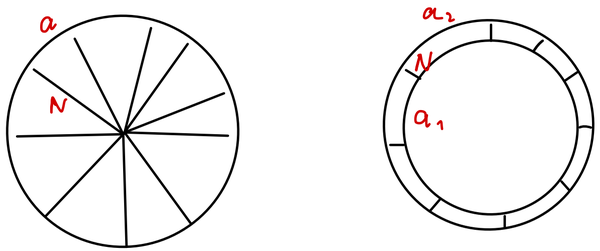

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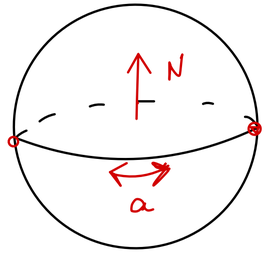
Compactified Lorentzian de Sitter



[Continuum:

Banihashemi, Jacobson]

Aim: Entropy from Lorentzian

Path integral.

With symmetry reduction:

Two integration variables ~ lapse, equator

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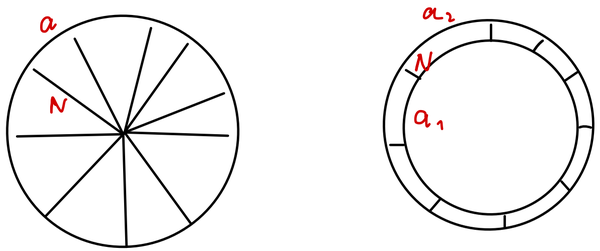

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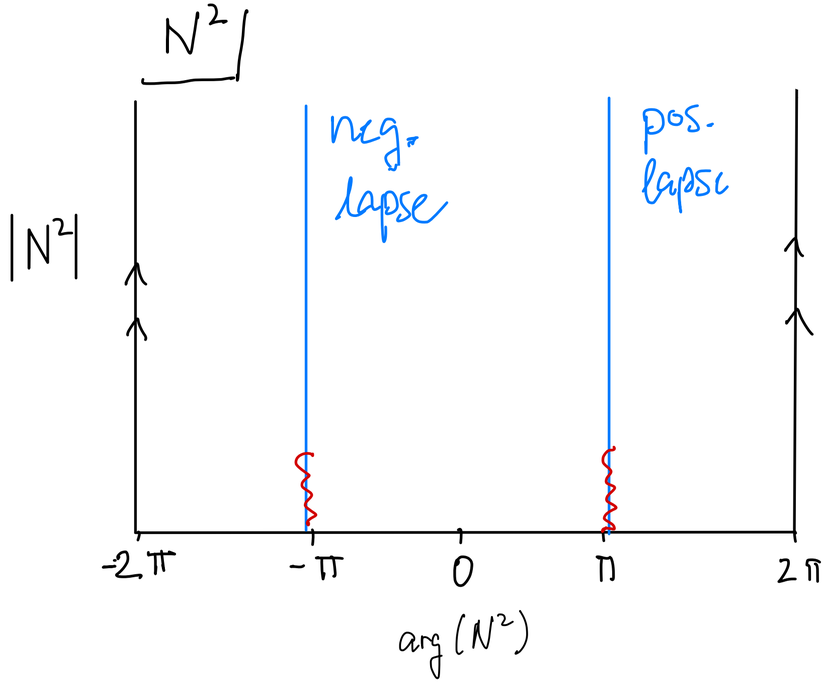

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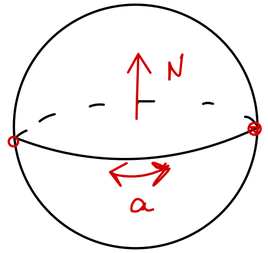

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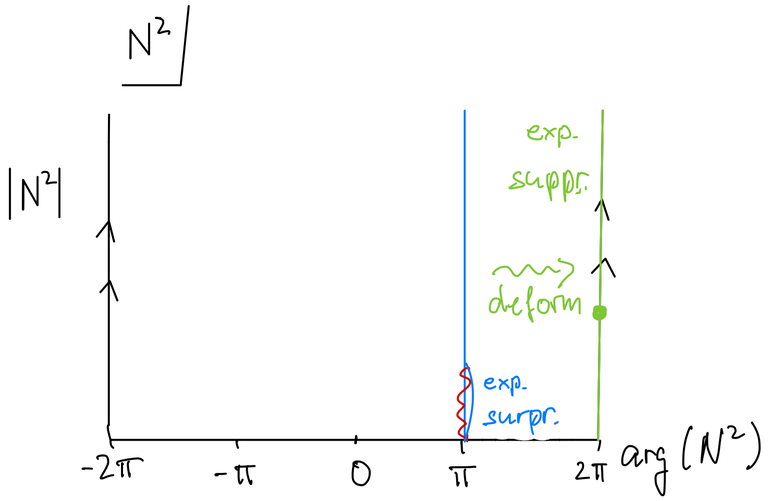

[Continuum-Mini-Super-Space:

Feldbrugge, Lehners, Turok]

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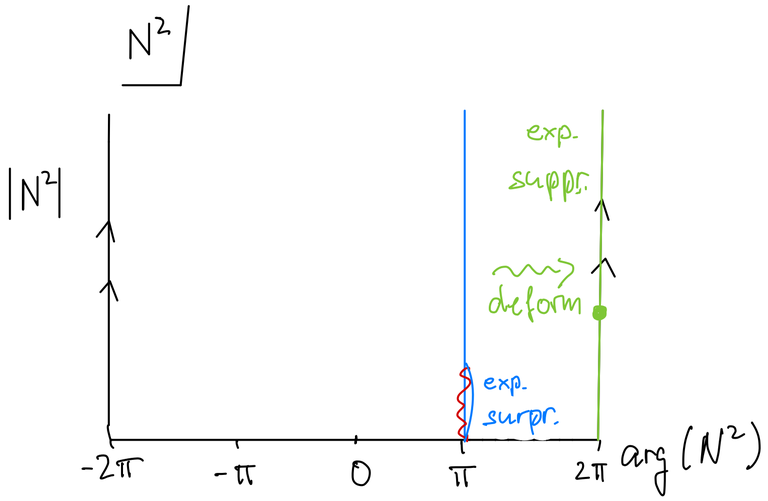

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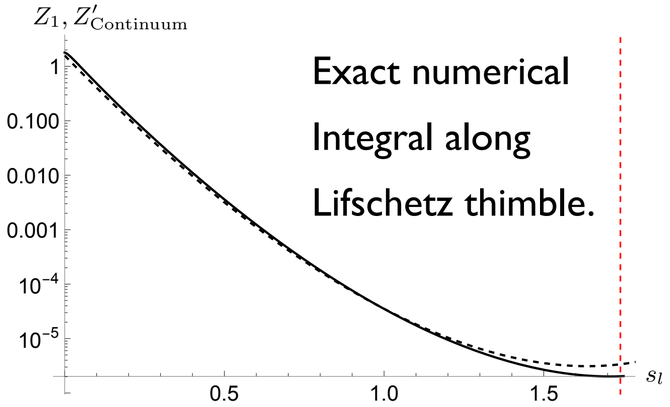

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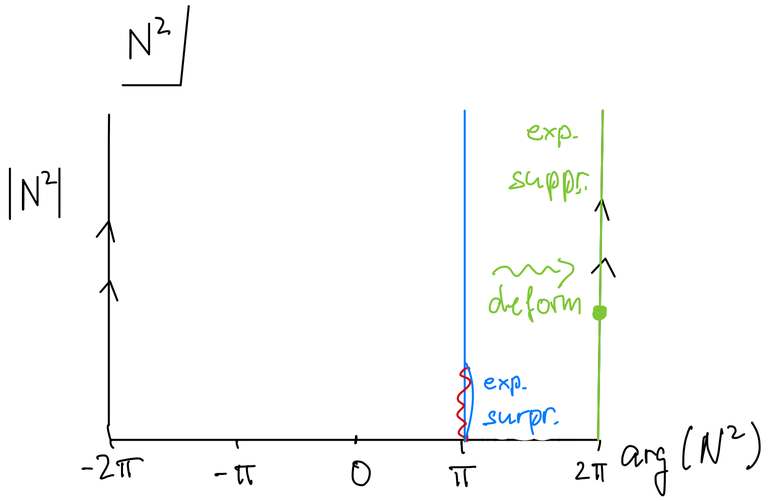

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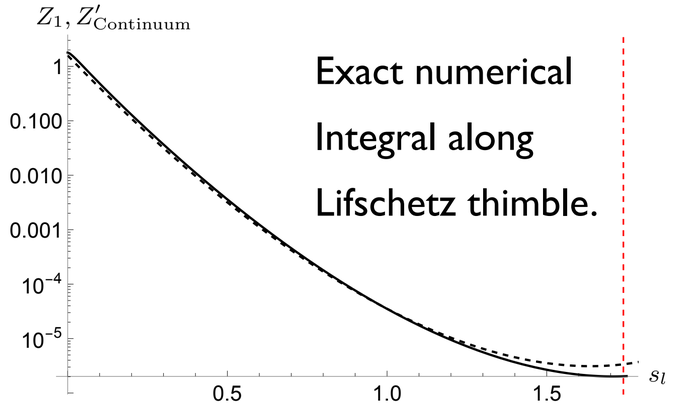

Need irregular configurations to agree with continuum result.

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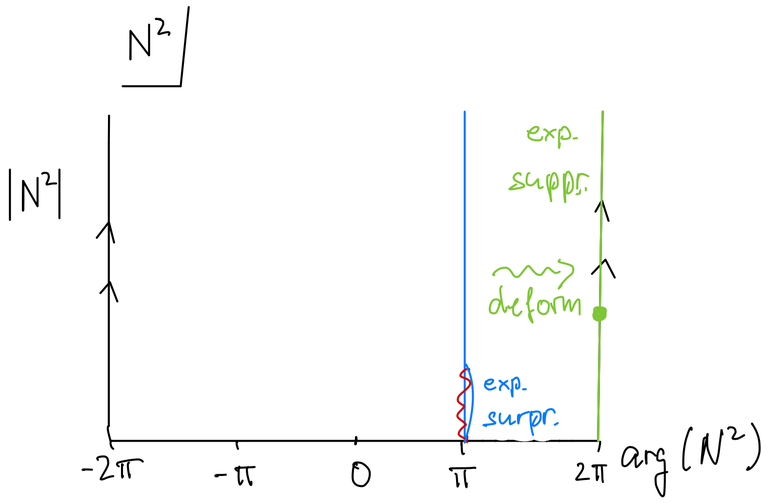

[Continuum-Mini-Super-Space:

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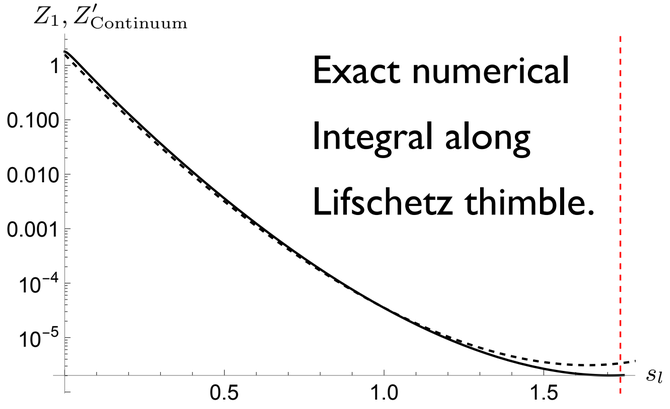

Exact numerical

Integral along

Lifschetz thimble.

Need irregular configurations to agree with continuum result.

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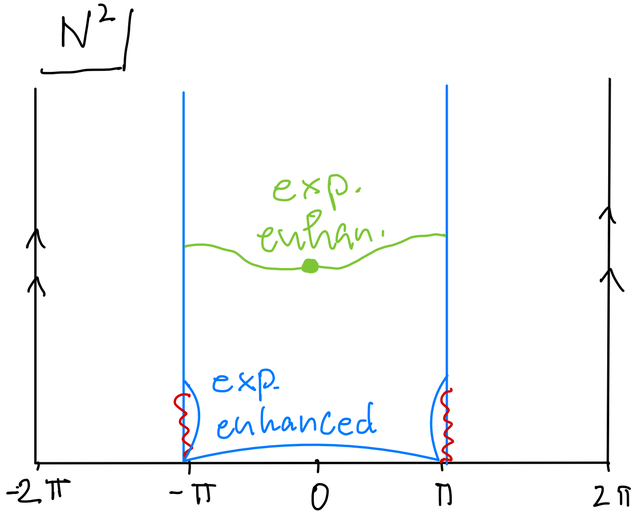

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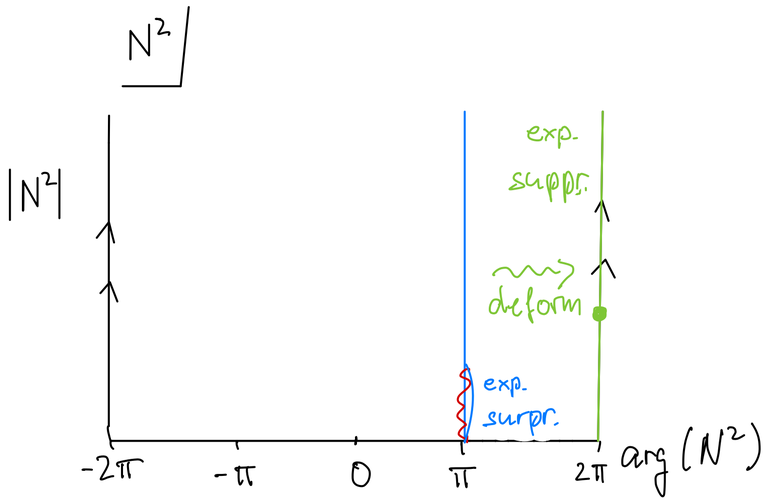

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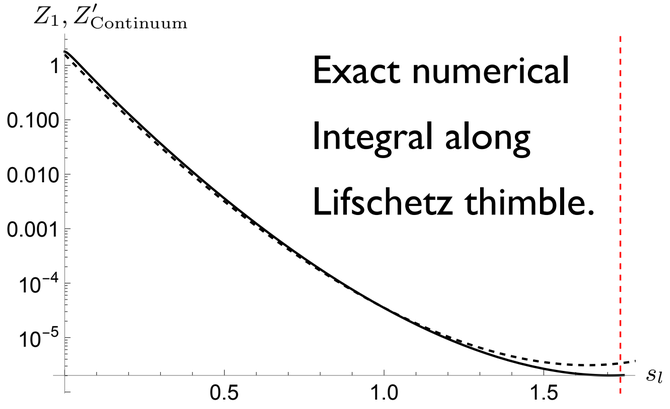

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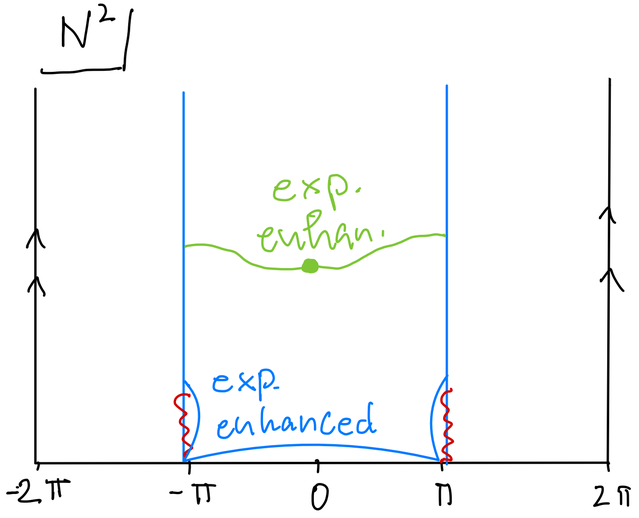

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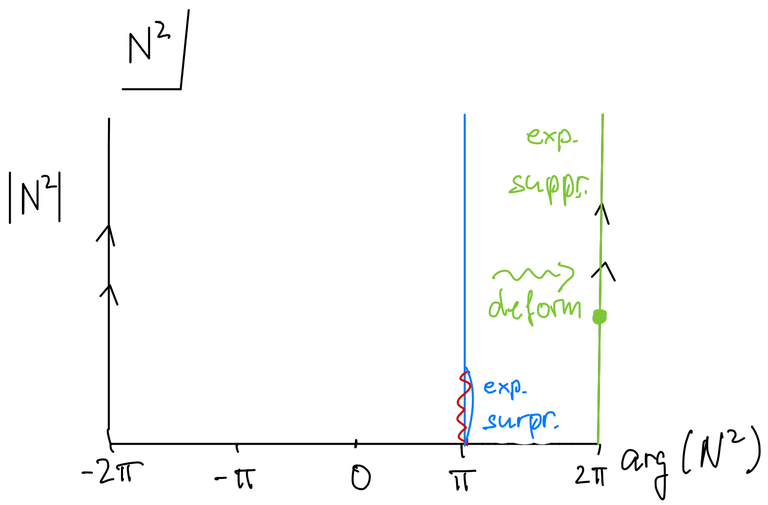

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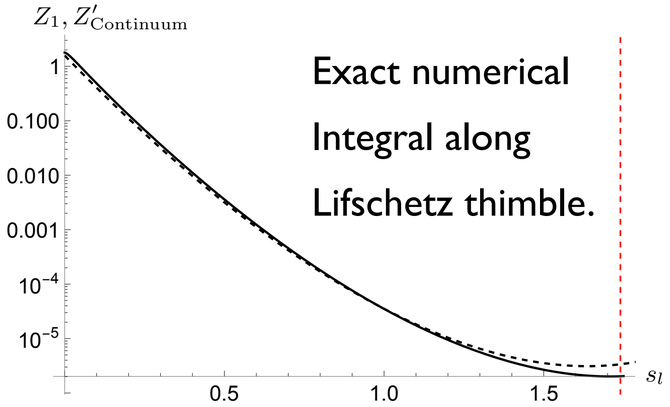

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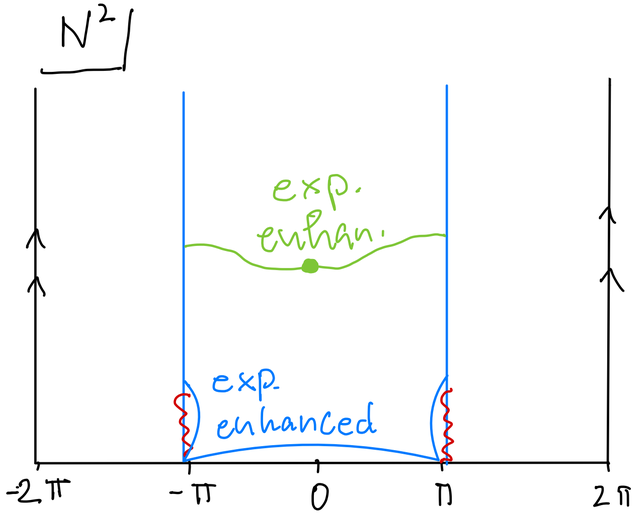

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Irregular light cone structures have an important role.
Surprise: important for entropy calculation.

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- Monte Carlo: not available.
- For integrals: deformation of contour, for example Lefschetz thimble.
- For sums? E.g. Spin foam sums.

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In particular: Shanks transform (with Wynn's epsilon algorithm).

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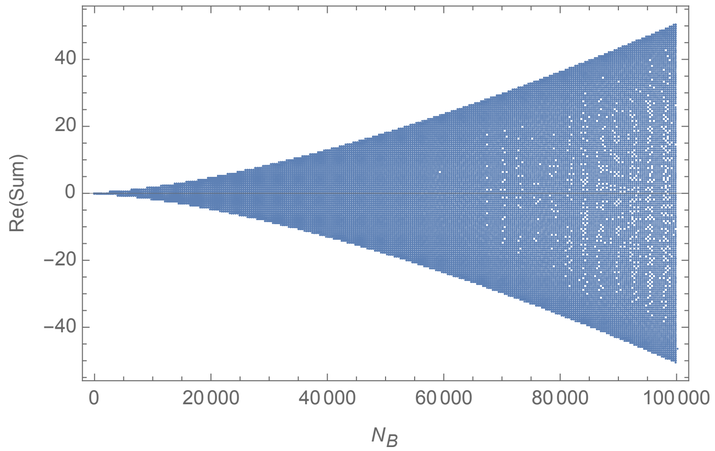
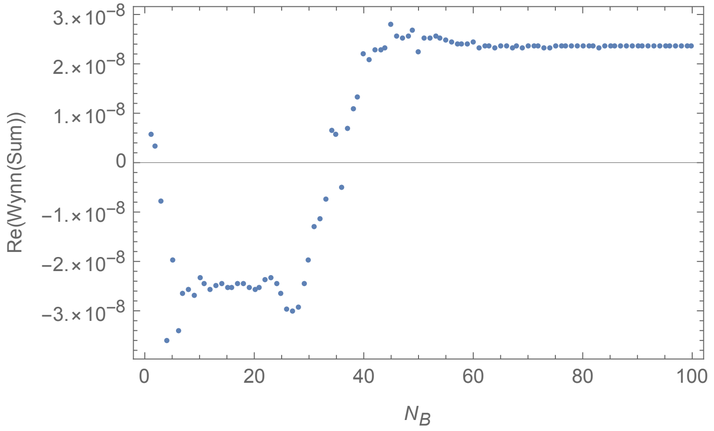
Effective spin foam Expectation values:





Rel. Error~10^{-8}

Works very well for sums with actions that are at most linear in the summation variable.
Consistent with quantum mechanics (Bohr quantization) and spin foams.

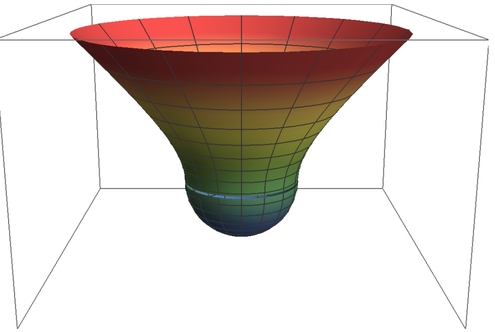
Effective spin foams for de Sitter

[BD, Padua-Arguelles 2023]

Due to symmetry reduction only difference between Regge path integral and effective spin foams: integral vs sum.

What is the effect of the discrete area spectrum?

Quantum deSitter



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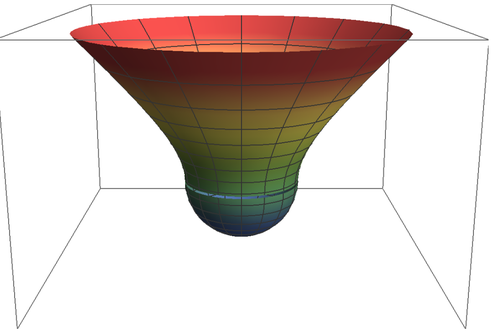

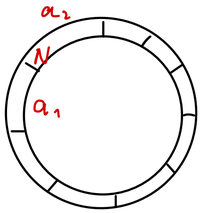

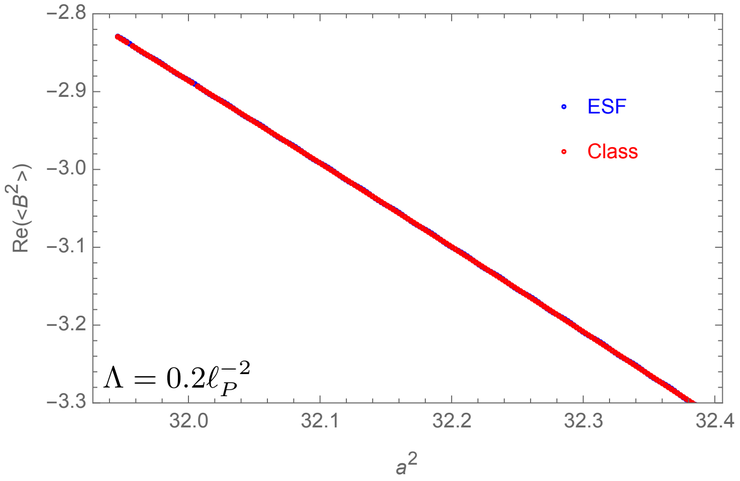

Lorentzian regime transition: very small quantum effects.
Almost no difference between Regge and spin foam.

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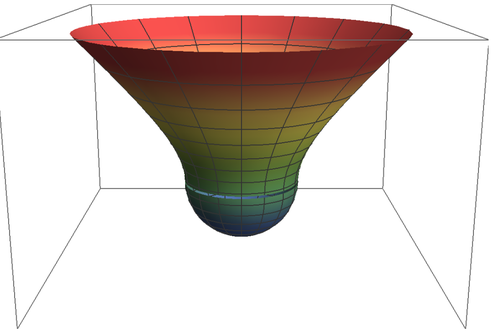

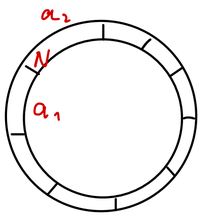

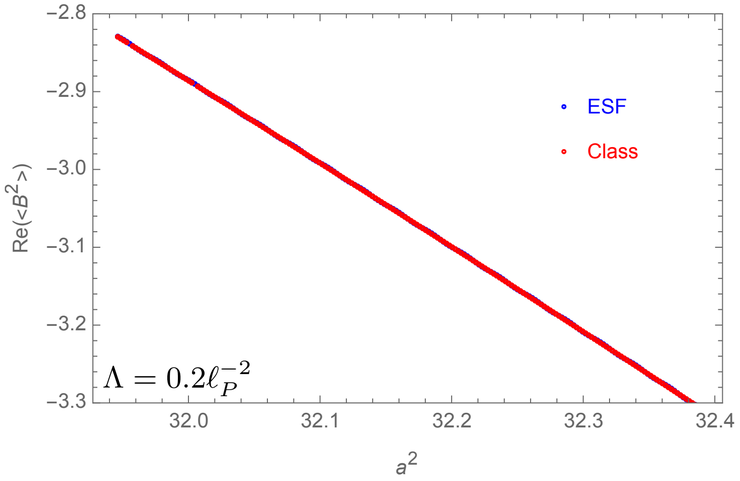

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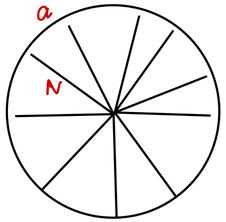

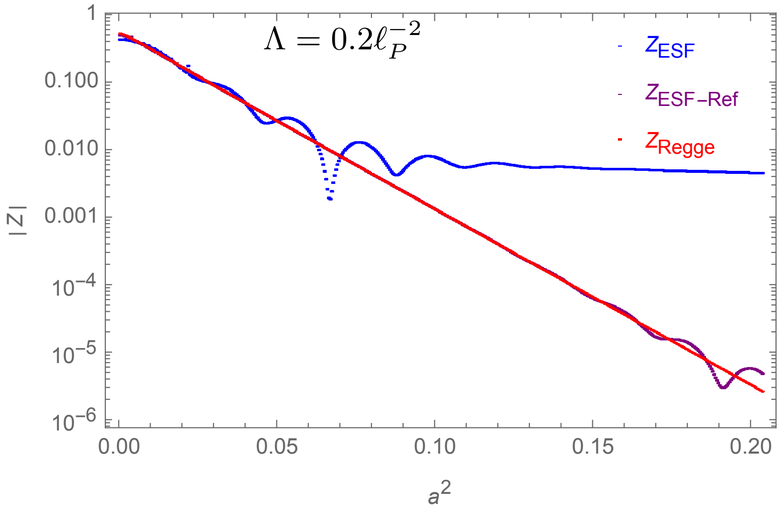

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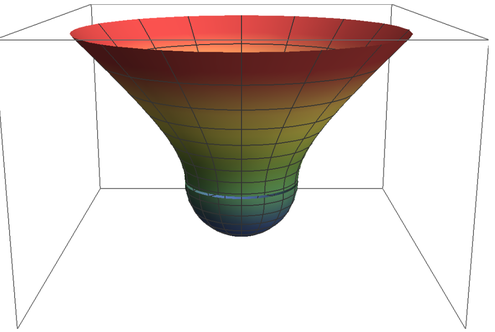

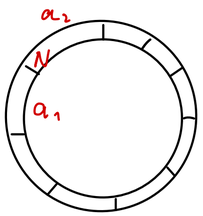

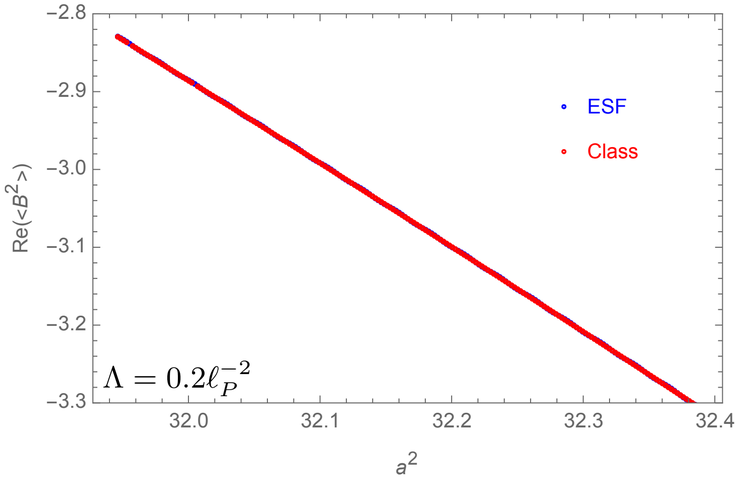

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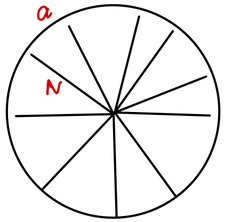

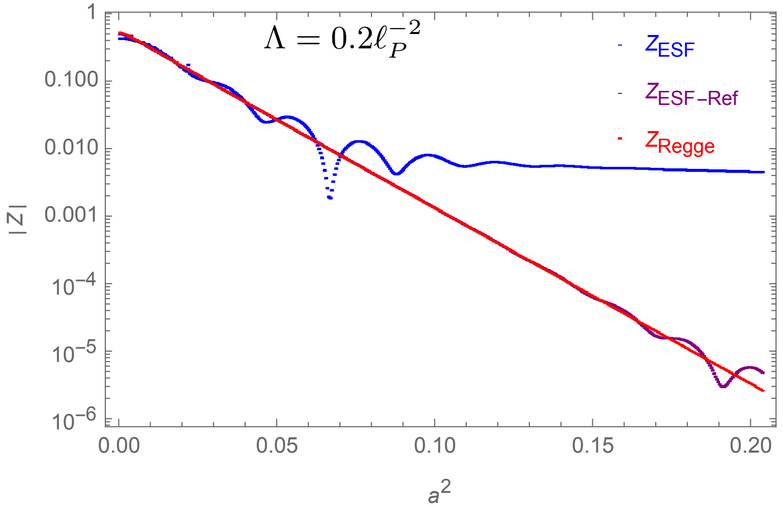

Euclidean regime transition:
Significant differences between Regge and spin foams.

Discrete spectra: Make tunnelling amplitudes less suppressed.

Should be confirmed by using more time steps.

Summary

- Continuum limit of spin foams can lead to general relativity.
- Effective action from spin foams: Weyl curvature squared term from quantum extension of configuration space to area metrics
- Ghost free

$$L_{eff} = L_{EH} - \frac{1}{4}\frac{1}{\Box - M(\gamma)^2}\text{Weyl}^2$$

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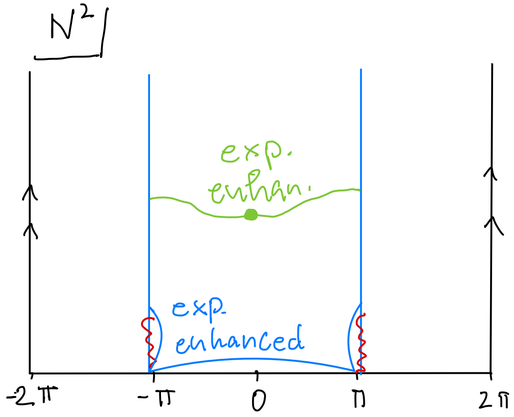
- Important role for thermodynamic interpretation
- Appears in the continuum in a much more subtle way



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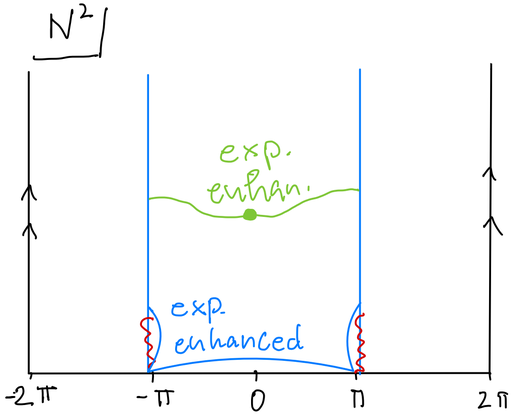
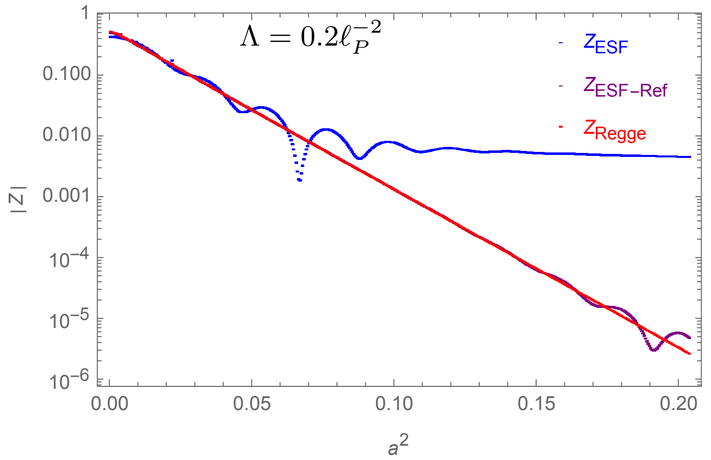


- Effective spin foam path integral for de Sitter
- Shank transform to deal with sums (and integrals)
- Weakening of decay of no-boundary probability amplitude



Lorentzian path integral

Spin foams, simplicial Regge, continuum, CDT, causal sets, …

Lots of things to understand!

Computational challenges …

Intriguing conceptual questions.