Lessons of the EFT Treatment of Quantum General Relativity

John F. Donoghue

1) EFT in a nutshell

2) A couple of examples

3) Seven Lessons of the EFT

4) Limits/limitations

AMHERST CENTER FOR FUNDAMENTAL INTERACTIONS
Physics at the interface: Energy, Intensity, and Cosmic frontiers
University of Massachusetts Amherst

Perimeter Institute
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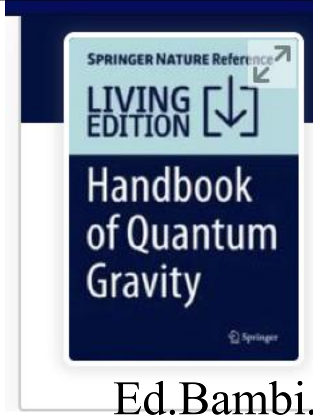
Varieties of EFTs in Gravity



Ed.Bambi, Modesto, Shapiro

Quantum General Relativity and Effective Field Theory

John F. Donoghue (Massachusetts U., Amherst) (Nov 17, 2022)

Effective Field Theory for Large-Scale Structure

Mikhail M. Ivanov (Princeton, Inst. Advanced Study) (Dec 16, 2022)

EFT for de Sitter Space

Daniel Green (UC, San Diego) (Oct 11, 2022)

Effective Field Theory for Compact Binary Dynamics

Walter D. Goldberger (Yale U.) (Dec 13, 2022)

Effective Field Theory and Applications: Weak Field Observables from Scattering Amplitudes in Quantum Field Theory

N. Emil J. Bjerrum-Bohr (Bohr Inst.), Ludovic Planté (Bohr Inst.), Pierre Vanhove (IPhT, Saclay) (Dec 17, 2022)

Gravity, Horizons and Open EFTs

C.P. Burgess (McMaster U. and Perimeter Inst. Theor. Phys. and CERN and Dublin Inst.), Greg Kaplanek (Imperial Coll., London) (Dec 18, 2022)

Soft-Collinear Gravity and Soft Theorems

Martin Beneke (Munich, Tech. U.), Patrick Hager (Munich, Tech. U.), Robert Szafron (Brookhaven) (

What is behind the EFT?

Low energy symmetry and fields

- general covariance and the metric

Uncertainty principle

- unknown physics at high energy => local

Path Integral with limits

$$Z^{core} = \int [d\phi d\psi dA dg]_{\text{Limits}} \exp\left[i\int d^4x\sqrt{-g}\left(-\frac{1}{4}g^{\mu\alpha}g^{\nu\beta}F^a_{\mu\nu}F^a_{\alpha\beta} + \ldots\text{SM}\ldots \right.\right.$$
$$\left.\left. - \Lambda_{cc} + \frac{2}{\kappa^2}R + c_1 R^2 + c_2 R_{\mu\nu}R^{\mu\nu}\ldots\right)\right]$$

- "Limits" describes the limitations of our understanding
- metric must be part of the PI

EFT techniques

Quantum methods sample all energies
- including where the EFT is incorrect

But wrong part is local => like parameters in Lagrangian
- calculations must respect symmetries (~ dim. reg.)
- match or measure parameters

Nonlocal parts are reliable
- only from low energy D.O.F. and interactions
- long distance propagation

In calculations near Minkowski:
- nonanalytic only from nonlocal

$$(q^2)^n \to \Box^n \delta(x)$$
$$\log(-q^2) \to L(x-y) = \langle x | \log \Box | y \rangle$$

Example 1: Corrections to the gravitational potential

Scattering potential

$$\begin{aligned}\langle f|T|i\rangle &\equiv (2\pi)^4\delta^{(4)}(p-p')(\mathcal{M}(q)) \\ &= -(2\pi)\delta(E-E')\langle f|\tilde{V}(\mathbf{q})|i\rangle\end{aligned}$$

Full result is the full scattering amplitude
NR Potential is a useful way of illustrating result

$$V(\mathbf{x}) = \frac{1}{2m_1}\frac{1}{2m_2}\int\frac{d^3q}{(2\pi)^3}e^{i\mathbf{q}\cdot\mathbf{x}}\mathcal{M}(\vec{q})$$

What to expect:

Momentum space amplitudes:

$$V(q^2) = \frac{GMm}{q^2}\left[1 + a'G(M+m)\sqrt{-q^2} + b'G\hbar q^2 \ln(-q^2) + c'Gq^2\right]$$

Non-analytic

Relation to position space:

$$\int \frac{d^3q}{(2\pi)^3}\ e^{i\mathbf{q}\cdot\mathbf{r}}\frac{1}{|\mathbf{q}|^2} = \frac{1}{4\pi r}$$

$$\int \frac{d^3q}{(2\pi)^3}\ e^{i\mathbf{q}\cdot\mathbf{r}}\frac{1}{|\mathbf{q}|} = \frac{1}{2\pi^2 r^2}$$

$$\int \frac{d^3q}{(2\pi)^3}\ e^{i\mathbf{q}\cdot\mathbf{r}}\ln(\mathbf{q}^2) = \frac{-1}{2\pi r^3}$$

General expansion:

$$V(r) = -\frac{GMm}{r}\left[1 + a\frac{G(M+m)}{rc^2} + b\frac{G\hbar}{r^2c^3}\right] + cG^2Mm\delta^3(r)$$

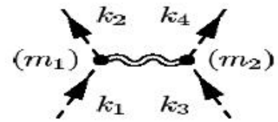
Classical quantum

Result:

:

$$\boxed{V(r) = -\frac{Gm_1m_2}{r}\left[1+3\frac{G(m_1+m_2)}{r}+\frac{41}{10\pi}\frac{G\hbar}{r^2}\right]}$$

Lowest order:



Vertex corrections:

Vacuum polarization:
(Duff 1974)

Box and crossed box

Others:

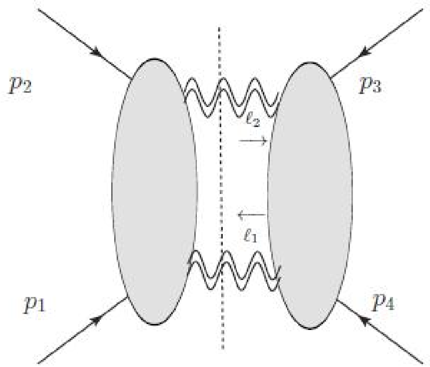
On-shell techniques and loops from unitarity

- On-shell amplitudes only
- No ghosts needed – axial gauge
- Exhibits "double copy" relations
- Both unitarity cuts and dispersion relation methods

$$iM^{1-\text{loop}}\Big|_{disc} = \int \frac{d^D\ell}{(2\pi)^D} \frac{\sum_{\lambda_1,\lambda_2} M^{\text{tree}}_{\lambda_1\lambda_2}(p_1,p_2,-\ell_2^{\lambda_2},\ell_1^{\lambda_1})(M^{\text{tree}}_{\lambda_1\lambda_2}(p_3,p_4,\ell_2^{\lambda_2},-\ell_1^{\lambda_1}))^*}{\ell_1^2\ell_2^2}\Big|_{cut},$$

$$iM_0^{\text{tree}}(p_1,p_2,k_1^+,k_2^+) = \frac{\kappa_{(4)}^2}{16}\frac{1}{(k_1\cdot k_2)}\frac{m^4\,[k_1\,k_2]^4}{(k_1\cdot p_1)(k_1\cdot p_2)},$$

$$iM_0^{\text{tree}}(p_1,p_2,k_1^-,k_2^+) = \frac{\kappa_{(4)}^2}{16}\frac{1}{(k_1\cdot k_2)}\frac{\langle k_1|p_1|k_2]^2\,\langle k_1|p_2|k_2]^2}{(k_1\cdot p_1)(k_1\cdot p_2)},$$

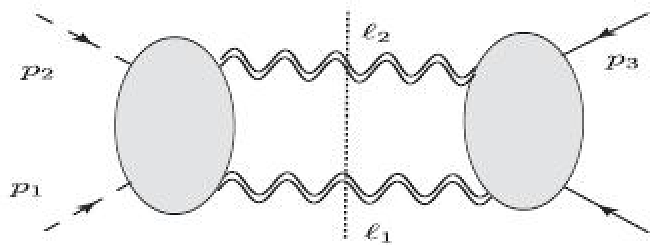


Confirm results for gravitational potential

- gauge invariance check

Example 2: Light bending at one loop

Again using unitarity methods



$$i\mathcal{M}^{[\eta(p_1)\eta(p_2)]}_{[\phi(p_3)\phi(p_4)]} \simeq \frac{\mathcal{N}^\eta}{\hbar}(M\omega)^2\left[\frac{\kappa^2}{t}+\kappa^4\frac{15}{512}\frac{M}{\sqrt{-t}}+\hbar\kappa^4\frac{15}{512\pi^2}\times\log\left(\frac{-t}{M^2}\right)-\hbar\kappa^4\frac{bu^\eta}{(8\pi)^2}\log\left(\frac{-t}{\mu^2}\right)+\hbar\kappa^4\frac{3}{128\pi^2}\log^2\left(\frac{-t}{\mu^2}\right)+\kappa^4\frac{M\omega}{8\pi}\frac{i}{t}\log\left(\frac{-t}{M^2}\right)\right], \qquad (11)$$

Can convert amplitude to bending angle using eikonal method

Result different for scalars, photons and gravitons

$$\theta = \frac{4GM}{b} + \frac{15}{4}\frac{G^2M^2\pi}{b^2} + \frac{8\mathrm{bu}^\eta - 47 + 64\log\frac{2r_0}{b}}{\pi}\frac{G^2\hbar M}{b^3}$$

with

$\mathrm{bu}^\eta = (371/120,\ 113/120,\ -29/8)$ for (scalar photons gravitons)

Seven Lessons of the EFT

1) Universality of the NR gravitational interaction

2) Classical physics from loops

3) No "test particle" limit for quantum effects

4) "Quantum corrected metric" is not a valid quantum concept

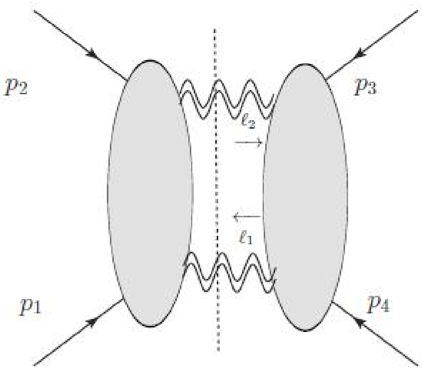
5) Trajectories of massless particles are not universal

6) CC and G are not running parameters

7) Lightcones/ Penrose diagrams etc likely uncontrolled approximations

1) Universality of the NR Gravitational Interaction

Soft theorems extend to some loop effects



Recall on-shell unitarity method

On-shell amplitudes satisfy soft theorems
- Low, Weinberg and Gross-Jackiw

The relevant cuts are exactly these universal pieces

Then the leading loop results are also universal
- first found painfully by Holstein and Ross
- then true for particles, molecules, the Moon etc.

2) Classical physics from loops

Folk theorem – the loop expansion is the $\hbar$ expansion

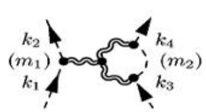
- not true
- classical physics also present in loop expansion
- hidden factors of hbar

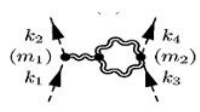
$$\mathcal{L} = \hbar\bar{\psi}\left(i\partial\!\!\!/ - \frac{m}{\hbar}\right)\psi$$

- at one loop, present in $\sqrt{q^2}$ non-analyticity

$$\sqrt{\frac{m^2}{-q^2}} \to \hbar\sqrt{\frac{m^2}{-\hbar^2 q^2}}$$

- both classical and quantum present in some diagrams





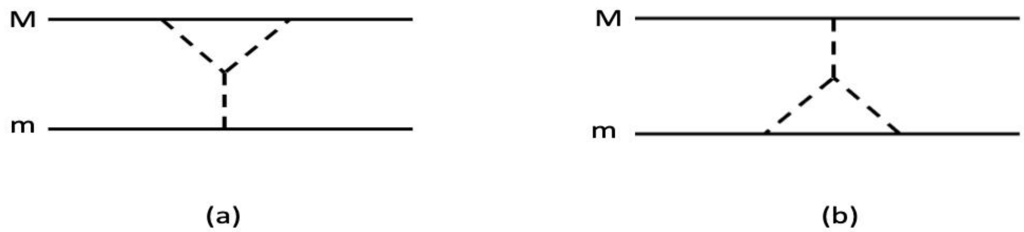
$$M_{5(a)+5(b)}(\vec{q}) = 2G^2 m_1 m_2 \left(\frac{\pi^2(m_1+m_2)}{|\vec{q}|} + \frac{5}{3}\log \vec{q}^2\right)$$

$$M_{5(c)+5(d)}(\vec{q}) = -\frac{52}{3}G^2 m_1 m_2 \log \vec{q}^2$$

This has become a vibrant subfield

3) There is no “test particle” limit for quantum effects

All quantum corrections are of the same form



Classical	$\frac{GM}{r}$	$\frac{Gm}{r}$	- test particle
Quantum	$\frac{G\hbar}{r^2}$	$\frac{G\hbar}{r^2}$	- no test particle possible

Also visible in the results for massless particle scattering

4) "Quantum corrected metric" is not a valid quantum item

Tempting to ask for eg. "quantum corrections to Schwarzschild"
- mea culpa

But not a well-defined quantum question

Specific objection– not field redefinition independent (Kirilin)

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu} \quad \rightarrow \quad g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu} + a h_{\mu\lambda} h^{\lambda}_{\nu}$$

- explicit calculation to demonstrate this

Haag's theorem only guarantees field redefinition independence
for **on-shell** matrix elements

Metric is only part of a full quantum calculation

5) Trajectories of massless particles are not universal

Recall:

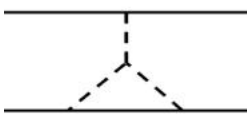
$$\theta = \frac{4GM}{b} + \frac{15}{4}\frac{G^2M^2\pi}{b^2} + \frac{8\mathrm{bu}^\eta - 47 + 64\log\frac{2r_0}{b}}{\pi}\frac{G^2\hbar M}{b^3}$$

with

$\mathrm{bu}^\eta = (371/120,\ 113/120,\ -29/8)$ for scalars, photons, gravitons

The quantum corrections amount to tidal forces

-long range propagation



- sample gravitational fields at more than one position

Not geodesic motion

6) Cosmological constant and G are not running parameters

-at least in EFT region

Most obviously – **no power-law running in physical processes**

- i.e. $\Lambda_{cc} \sim (\Lambda_{\text{cutoff}})^4 \qquad G \sim (\Lambda_{\text{cutoff}})^2$
- physical running with kinematic quantities $\sim q^2, R$
- energy expansion of Lagrangian
- no universal repackaging as running parameters

But also **not log running** with energy scale

- kinematic logs not related to renormalization of CC or R

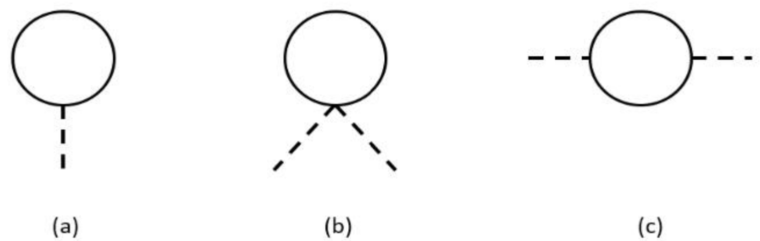
Some points:

a) Renormalizaton of CC and (non) running
b) Non-local effective actions
c) Non-local partners

a) Example: Renormalization of CC from massive particle

Can be probed by individual metric couplings

$$\sqrt{-g}\Lambda = \sqrt{-\bar{g}}\Lambda(1 + \frac{1}{2}h^{\sigma}_{\sigma} + \frac{1}{8}(h^{\sigma}_{\sigma})^2 - \frac{1}{4}h_{\sigma\lambda}h^{\sigma\lambda} + ...)$$



$$-i\mathcal{M} = i\frac{m^4}{32\pi^2}\left(\frac{1}{2}h + \frac{1}{8}h^2 - \frac{1}{4}h^{\mu\nu}h_{\mu\nu}\right)\left[\frac{1}{\bar{\epsilon}} + \log\frac{\mu^2}{m^2} + \frac{3}{2}\right]$$

Tadpole diagram can have no momentum flow through it

But also $\mu\frac{\partial\mathcal{M}}{\partial\mu} \neq 0$ does not imply physical running

No kinematic variable involved

Logarithm disappear when renormalized

b) Nonlocal effective actions and running

Example QED

$$S = \int d^4x \ -\frac{1}{4}F_{\rho\sigma}\left[\frac{1}{e^2(\mu)} + b_i \ln\left(\Box/\mu^2\right)\right]F^{\rho\sigma}$$

With

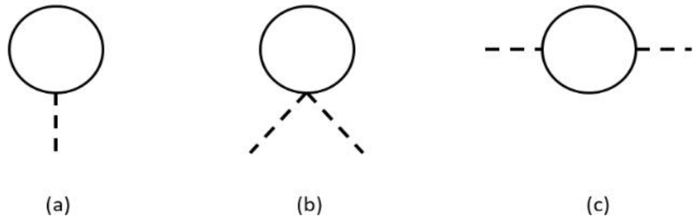
$$\langle x|\ln\left(\frac{\Box}{\mu^2}\right)|y\rangle \equiv L(x-y) = \int \frac{d^4q}{(2\pi)^4} e^{-iq\cdot(x-y)} \ln\left(\frac{-q^2}{\mu^2}\right) \quad .$$

There is true running in gravity at order $\boldsymbol{R^2}$ (Barvinsky Vilkovisky)

$$S \sim \int d^4x\sqrt{-g}\left[... + c_1(\mu_R)R^2 + b_1 R\log(\Box/\mu_R^2)R + ...\right]$$

But these constructions do not work with CC and R

i.e. $\frac{2}{\kappa^2}R + b\Gamma_\mu \log(\Box/\mu_R^2)\Gamma^\mu$ is not covariant



c) Non-local "partners"

There are residual energy scale dependences
- starting at order h^2

$$\mathcal{M}_{\mu\nu\alpha\beta} = \frac{1}{160\pi^2 q^4} \left(Q_{\mu\nu}Q_{\alpha\beta} + Q_{\mu\alpha}Q_{\nu\beta} + Q_{\mu\beta}Q_{\nu\alpha}\right) \left[m^4 J(q^2) + \frac{1}{6} m^2 q^2 - 3m^2 q^2 J(q^2)\right]$$

with $Q_{\mu\nu} = q_\mu q_\nu - \eta_{\mu\nu} q^2$ and $J(q^2) = \int_0^1 dx \, \log\left[\frac{m^2 - x(1-x)q^2}{m^2}\right]$

This is zeroth order in the derivative expansion (like cc)
- but only active above the scale m

When completed ala Barvinsky Vilkovisky:

$$\begin{aligned}\mathcal{L} \;&=\; \frac{m^4}{40\pi^2}\left[\left(\frac{1}{\Box}R_{\lambda\sigma}\right)\log((\Box + m^2)/m^2)\left(\frac{1}{\Box}R^{\lambda\sigma}\right) - \frac{1}{8}\left(\frac{1}{\Box}R\right)\log((\Box + m^2)/m^2)\left(\frac{1}{\Box}R\right)\right] \\ &+\; \frac{m^2}{240\pi^2}\left[R_{\lambda\sigma}\frac{1}{\Box}R^{\lambda\sigma} - \frac{1}{8}R\frac{1}{\Box}R\right]\end{aligned}$$

7) Light cones etc likely uncontrolled approximations

Evident from bending calculations above

Corrections are tiny at low energy

But eventually become of order unity as EFT fails

Classical concepts seem to fail
- lightcones
- geodesics
- Penrose diagrams
- manifold structure
- causality ?

"Gravity is geometry" is a classical notion
- perhaps not best for the quantum theory

Limits of the EFT - High Energy

Expect GREFT to fail below or around M_P

- becomes strongly coupled $\frac{q^2}{M_P^2} \log q^2$

Example: QCD and Chiral Perturbation Theory

$$\Lambda_\chi \sim 0.6 \text{ GeV} \quad , \quad 4\pi F_\pi \sim 1.2 \text{ GeV} \quad , \quad \text{quark, gluon DOF} \sim 2 \text{ GeV}$$

But, parametrically decoupled

Full field theory encoded in coefficients

Example: ChPTh

$$L_\mu = U \partial_\mu U^\dagger$$

$$\mathcal{L} = \frac{F_\pi^2}{4} Tr(L_\mu L^\mu) + c_1 [Tr(L_\mu L^\mu)]^2 + c_2 Tr([L_\mu, L_\nu][L^\mu, L^\nu])$$

- linear sigma model $\quad c_1 \sim F_\pi^2/m_\sigma^2 \quad c_2 \sim 0$
- QCD $\quad c_1 \sim 0 \quad c_2 \sim F_\pi^2/m_\rho^2$

For GREFT,

Large c_1, c_2 implies lower energy breakdown

Limitations and Technical Challenges

But also low energy challenges

- basically gravity effects build up
- local terms use curvature expansion
- metric as variable
- metric grows between regions of small curvature
- nonlocal terms sample metric at distant points
- issue even for classical gravity

Not completely unique to gravity

- Skyrmions in chiral theories

But crucial for possibility of large quantum effects

Consider Reimann normal coordinates

Taylor expansion in a local neighborhood:

$$g_{\mu\nu}(y) = \eta_{\mu\nu} + \frac{1}{3}R_{\mu\alpha\nu\beta}(y_0)y^\alpha y^\beta - \frac{1}{6}R_{\mu\alpha\nu\beta;\gamma}(y_0)y^\alpha y^\beta y^\gamma$$
$$+ \left[\frac{1}{20}R_{\mu\alpha\nu\beta;\gamma\delta}(y_0) + \frac{2}{45}R_{\alpha\mu\beta\lambda}(y_0)R^\lambda{}_{\gamma\nu\delta}(y_0)\right] y^\alpha y^\beta y^\gamma y^\delta + \mathcal{O}(\partial^5)$$

Even for small curvature,
there is a limit to a perturbative treatment of long distance:

$$R_{\mu\alpha\nu\beta}(y_0)y^\alpha y^\beta << 1$$

Horizons are extreme example:

- locally safe – we could be passing a BH horizon right now
    - local neighborhood makes a fine EFT
    - can be small curvature

But quantum effects sample long distance

Recent work on classical BH and decoherence
- Danielson, Satishchandran, Wald
- issue for all quantum theories

EFT has some difficulties at long distances
- what is the parameter governing the problem?
- integrated curvature?

Summary:

Phrasing issue as "QM incompatible with GR" is misleading

GR is a very normal quantum EFT

There are lessons about quantum gravity here

But there are also limitations / technical challenges

$$Z^{core} = \int [d\phi d\psi dA dg]_{\text{Limits}} \exp\left[i \int d^4x \sqrt{-g} \left(-\frac{1}{4} g^{\mu\alpha} g^{\nu\beta} F^a_{\mu\nu} F^a_{\alpha\beta} + \ldots\text{SM}\ldots \right.\right.$$
$$\left.\left. - \Lambda_{cc} + \frac{2}{\kappa^2} R + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} \ldots \right) \right]$$