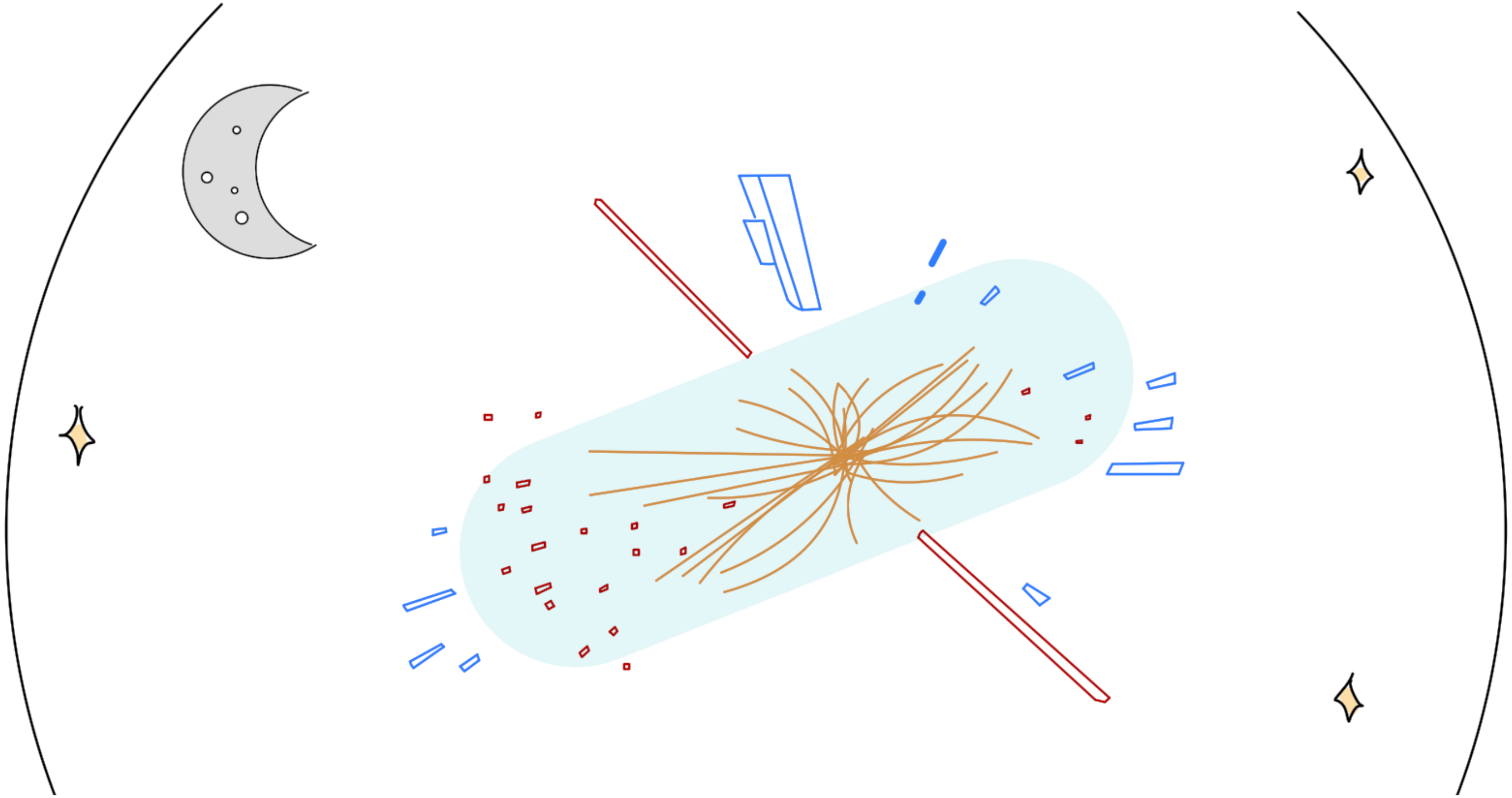
# Piecing Together a Flat Hologram

Sabrina Pasterski

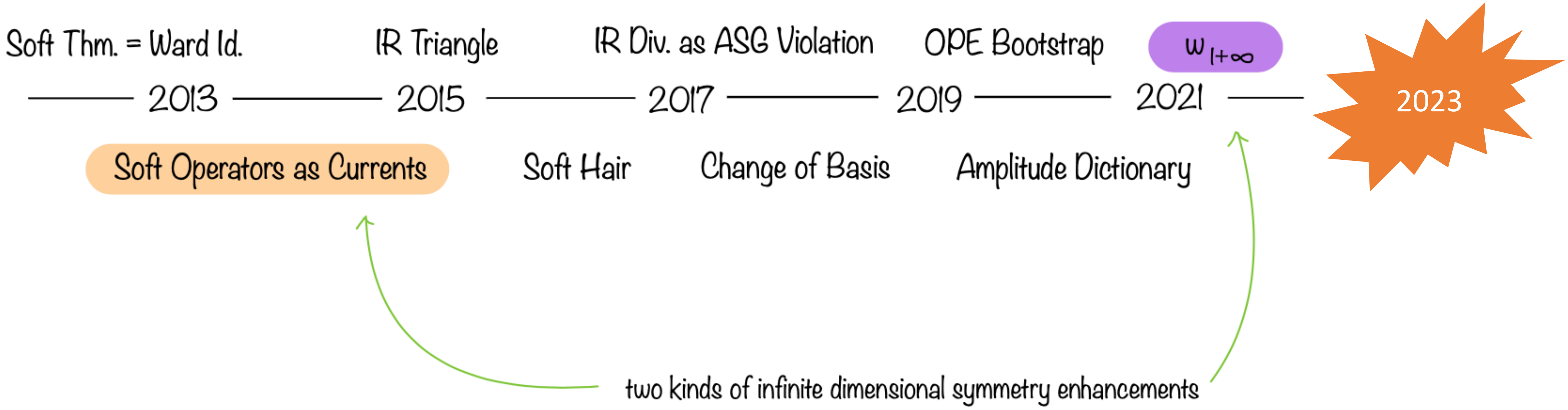
# Celestial Holography in 30 minutes

**The Celestial Conjecture:**

scattering in asymptotically flat spacetimes is dual to a CFT living on the celestial sphere

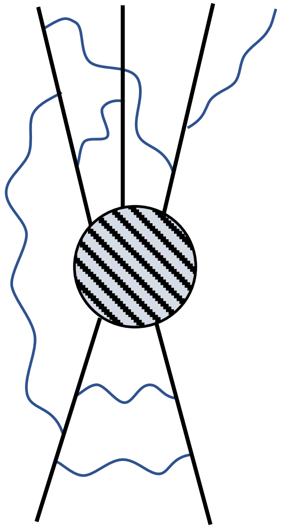


This program evolved from a **bottom-up** approach to flat holography…




… recognizing **soft theorems as Ward Identities** for asymptotic symmetries and recasting the **soft operators as currents** in a codimension 2 CFT.

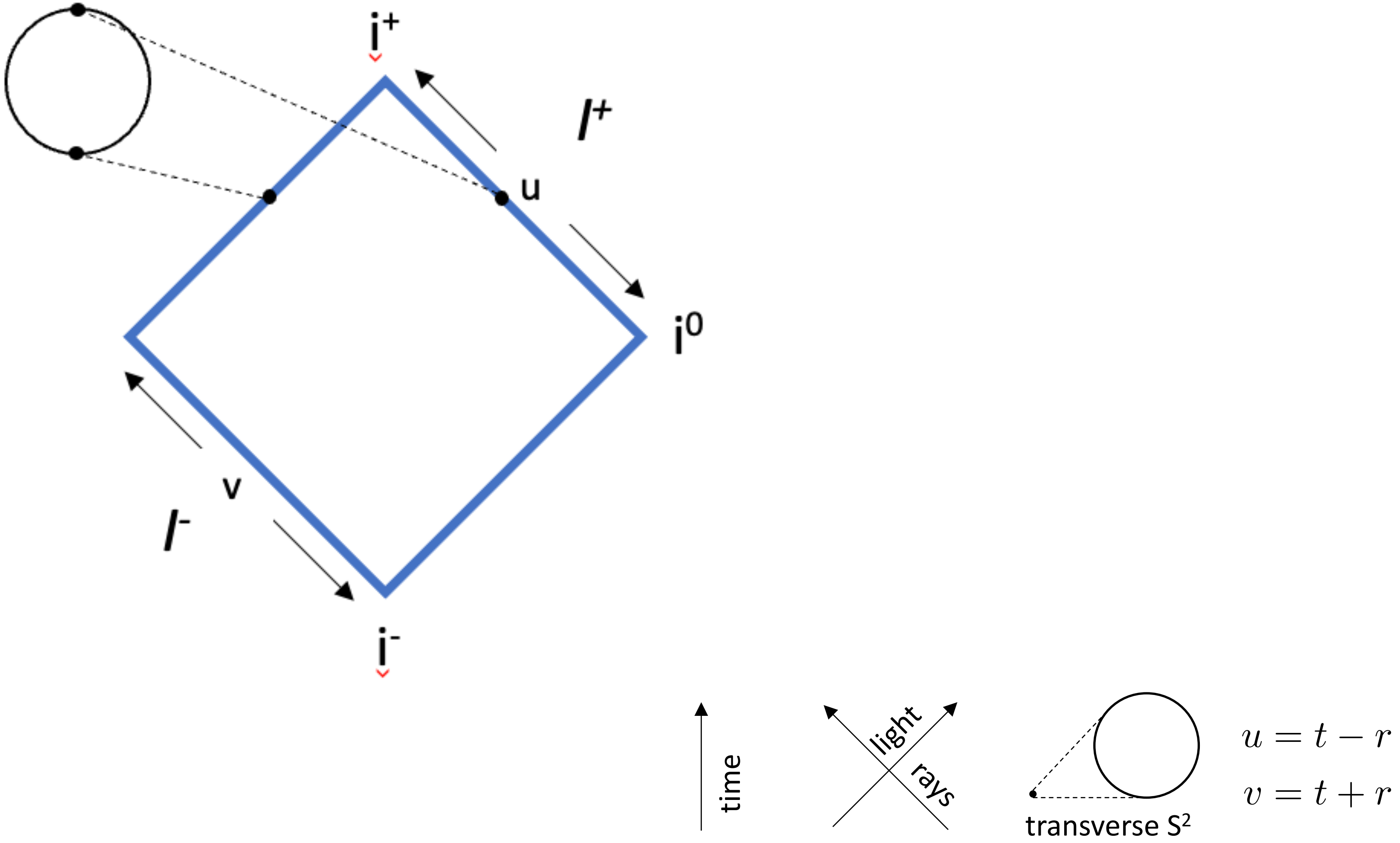
# Soft Thm = Ward Id



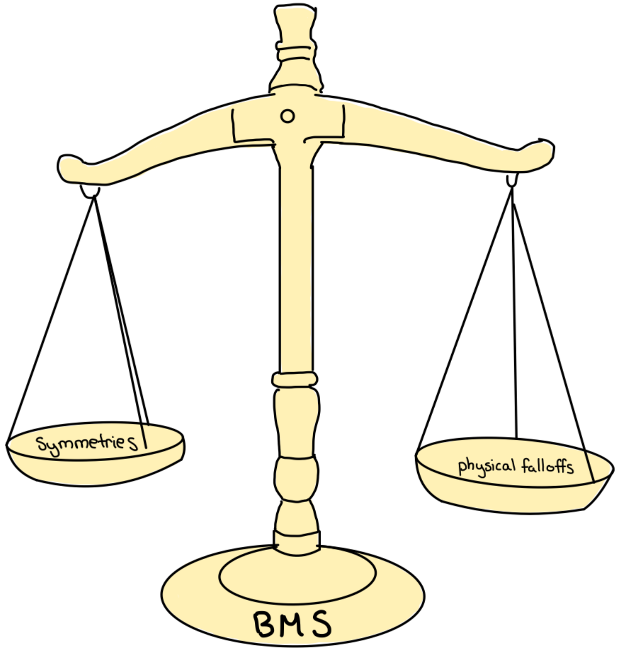
$$\Leftrightarrow \qquad \langle out|Q^+[Y]\mathcal{S} - \mathcal{S}Q^-[Y]|in\rangle = 0$$

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$$

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$



i+
I+
u
i0
v
I-
i-
time
light
rays
transverse S2
u = t − r
v = t + r




$$\text{ASG} = \frac{\text{allowed gauge symmetries}}{\text{trivial gauge symmetries}}$$

In Bondi gauge the metric near future null infinity takes the form

$$
\begin{aligned}
ds^2 \quad &= -du^2 - 2dudr + 2r^2\gamma_{z\bar{z}}dzd\bar{z} + 2\tfrac{m_B}{r}du^2 \\
&\underbrace{+\big(rC_{zz}}_{\text{Radiative Data}}dz^2 + D^zC_{zz}dudz + \tfrac{1}{r}(\tfrac{4}{3}N_z - \tfrac{1}{4}\partial_z(C_{zz}C^{zz}))dudz + c.c.\big) + \ldots
\end{aligned}
$$

which is preserved by the residual diffeomorphisms

$$
\begin{aligned}
\xi^+ =& \underbrace{(1+\frac{u}{2r})Y^{+z}\partial_z - \frac{u}{2r}D^{\bar{z}}D_zY^{+z}\partial_{\bar{z}} - \frac{1}{2}(u+r)D_zY^{+z}\partial_r + \frac{u}{2}D_zY^{+z}\partial_u + c.c}_{\text{Superrotations}} \\
& \underbrace{+ f^+\partial_u - \frac{1}{r}(D^zf^+\partial_z + D^{\bar{z}}f^+\partial_{\bar{z}}) + D^zD_zf^+\partial_r}_{\text{Supertranslations}}
\end{aligned}
$$

$$8\pi G Q^{+}[Y] = \int_{\mathcal{I}^{+}} \sqrt{\gamma} d^2 z du \ \left[ \underbrace{-\frac{1}{2} D_z^3 Y^z u \partial_u C^{zz}} + \underbrace{Y^z T_{uz} + u D_z Y^z T_{uu}} + h.c. \right]$$

$$Q^{+}[Y] = Q_S^{+}[Y] + Q_H^{+}[Y]$$

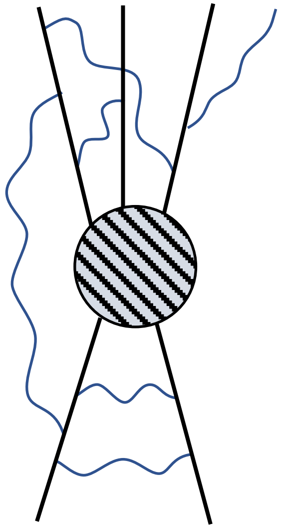
$$h_{\mu\nu}(x) = \sum_{\alpha=\pm} \int \frac{d^3q}{(2\pi)^3} \frac{1}{2\omega_q} \left[ \epsilon^{\alpha *}_{\mu\nu}(\vec{q}) a_\alpha(\vec{q}) e^{iq\cdot x} + \epsilon^{\alpha}_{\mu\nu}(\vec{q}) a_\alpha(\vec{q})^\dagger e^{-iq\cdot x} \right]$$

radiative mode

$$C_{\bar{z}\bar{z}} = 2 \lim_{r\to\infty} \frac{1}{r} \partial_{\bar{z}} x^\mu \partial_{\bar{z}} x^\nu \sum_{\alpha=\pm} \int \frac{d^3q}{(2\pi)^3} \frac{1}{2\omega_q} \left[ \epsilon^{\alpha *}_{\mu\nu}(\vec{q}) a_\alpha(\vec{q}) e^{-i\omega_q u - i\omega_q r(1-\cos\theta)} + h.c. \right]$$

saddle

$$C_{\bar{z}\bar{z}} = -\frac{i}{4\pi^2} \hat{\epsilon}^{+}_{\bar{z}\bar{z}} \int_0^\infty d\omega_q [a_-(\omega_q \hat{x}) e^{-i\omega_q u} - a_+(\omega_q \hat{x})^\dagger e^{i\omega_q u}]$$



$$\langle out|a_{-}(q)\mathcal{S}|in\rangle = \left(S^{(0)-} + S^{(1)-}\right)\langle out|\mathcal{S}|in\rangle + \mathcal{O}(\omega)$$

$$S^{(0)-} = \sum_{k} \frac{(p_k \cdot \epsilon^{-})^2}{p_k \cdot q} \qquad S^{(1)-} = -i\sum_{k} \frac{p_{k\mu}\epsilon^{-\mu\nu}q^{\lambda}J_{k\lambda\nu}}{p_k \cdot q}$$

# Soft Thm = Ward Id

$$\boxed{< out|Q^{+}[Y]\mathcal{S} - \mathcal{S}Q^{-}[Y]|in >= 0}$$

$$Q^{+}[Y] = Q^{+}_{S}[Y] + Q^{+}_{H}[Y]$$

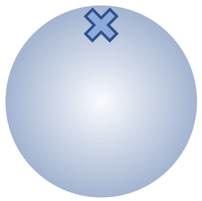
$$Q^{+}_{S}[Y] = \frac{1}{2}\int_{\mathcal{I}^{+}} dud^{2}zD^{3}_{z}Y^{z}u\partial_{u}C^{z}{}_{\bar{z}}$$

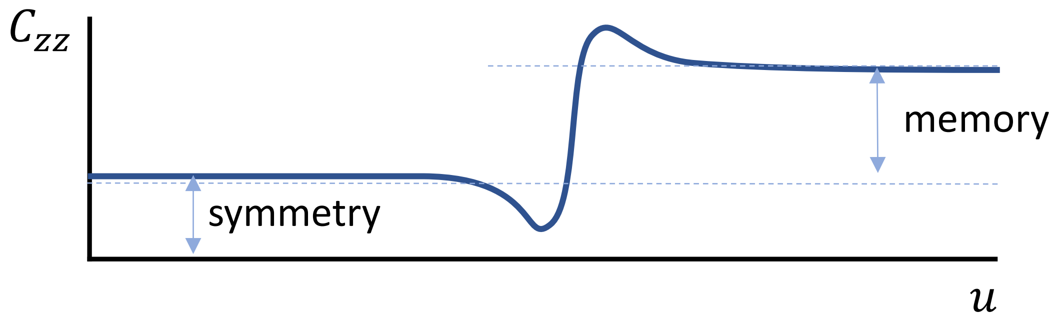
$$Q^{+}_{H}[Y] = \lim_{\Sigma\to\mathcal{I}^{+}}\int_{\Sigma} d\Sigma\ \xi^{\mu}n^{\nu}_{\Sigma}T^{M}_{\mu\nu}$$

$$\langle out|a_{-}(q)\mathcal{S}|in\rangle = \left(S^{(0)-} + S^{(1)-}\right)\langle out|\mathcal{S}|in\rangle + \mathcal{O}(\omega)$$

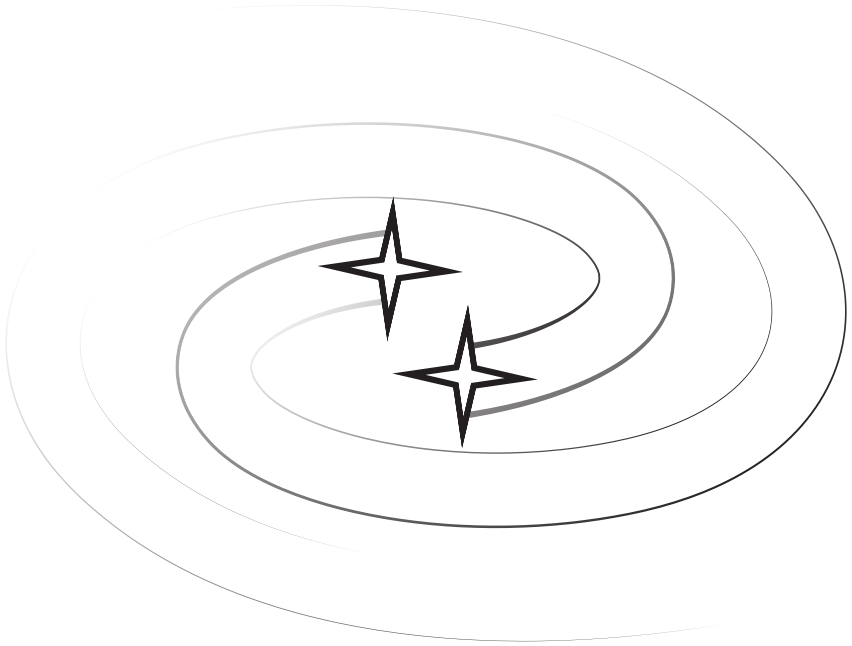
$$S^{(1)-} = -i\sum_{k}\frac{p_{k\mu}\epsilon^{-\mu\nu}q^{\lambda}J_{k\lambda\nu}}{p_{k}\cdot q}$$

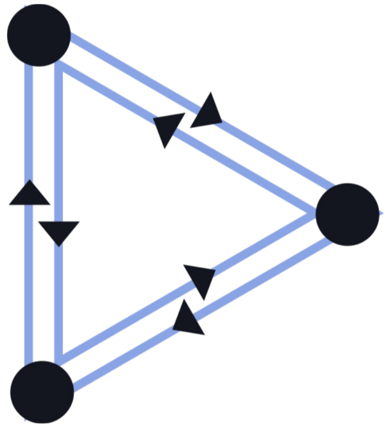
# Soft Thm = Memory





# Soft Thm = Memory

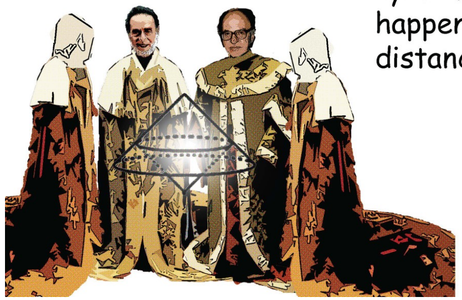


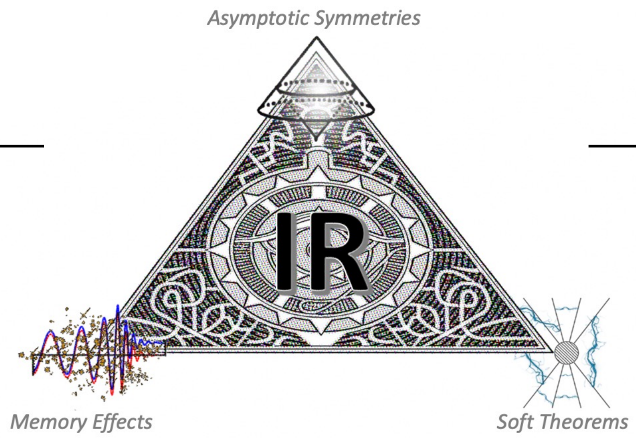


# IR Triangle

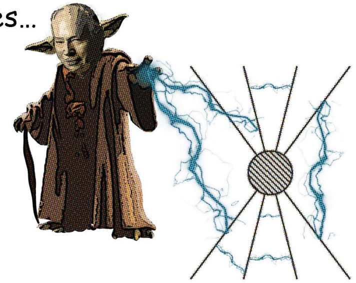
Our story starts with Strominger's suggestion that…


The relativists were systematizing what happens at long distances…






The quantum field theorists were worried about what was going on at low energies…



And, a little later, someone remembered there was a physical observable attached to each of these things….

# IR Triangle

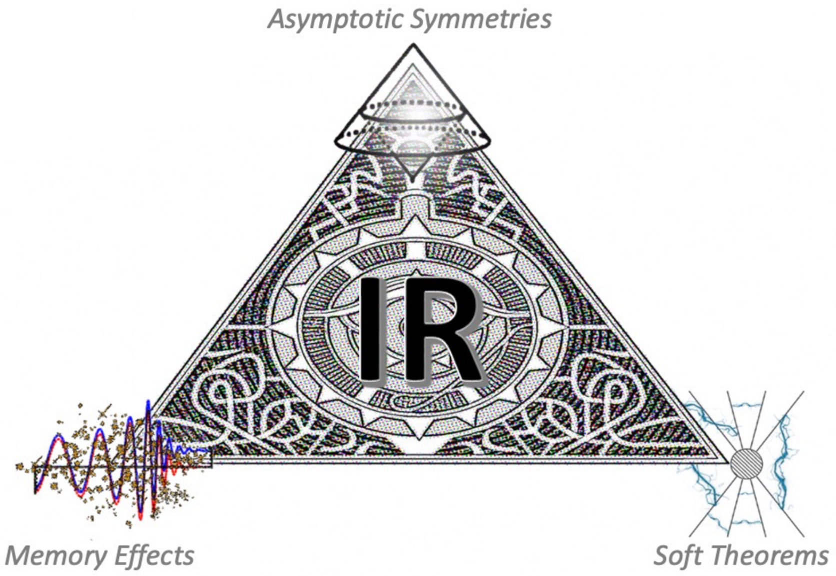
Boost Basis

2D Stress Tensor

Superrotation
Symmetry

New Subleading
Soft Graviton Thm.

Spin Memory



# 4D Soft Mode = 2D Current

For a particular choice of Y

$$T_{zz} = 2iQ_S^+(Y^w = \frac{1}{z-w}, Y^{\bar{w}} = 0)$$

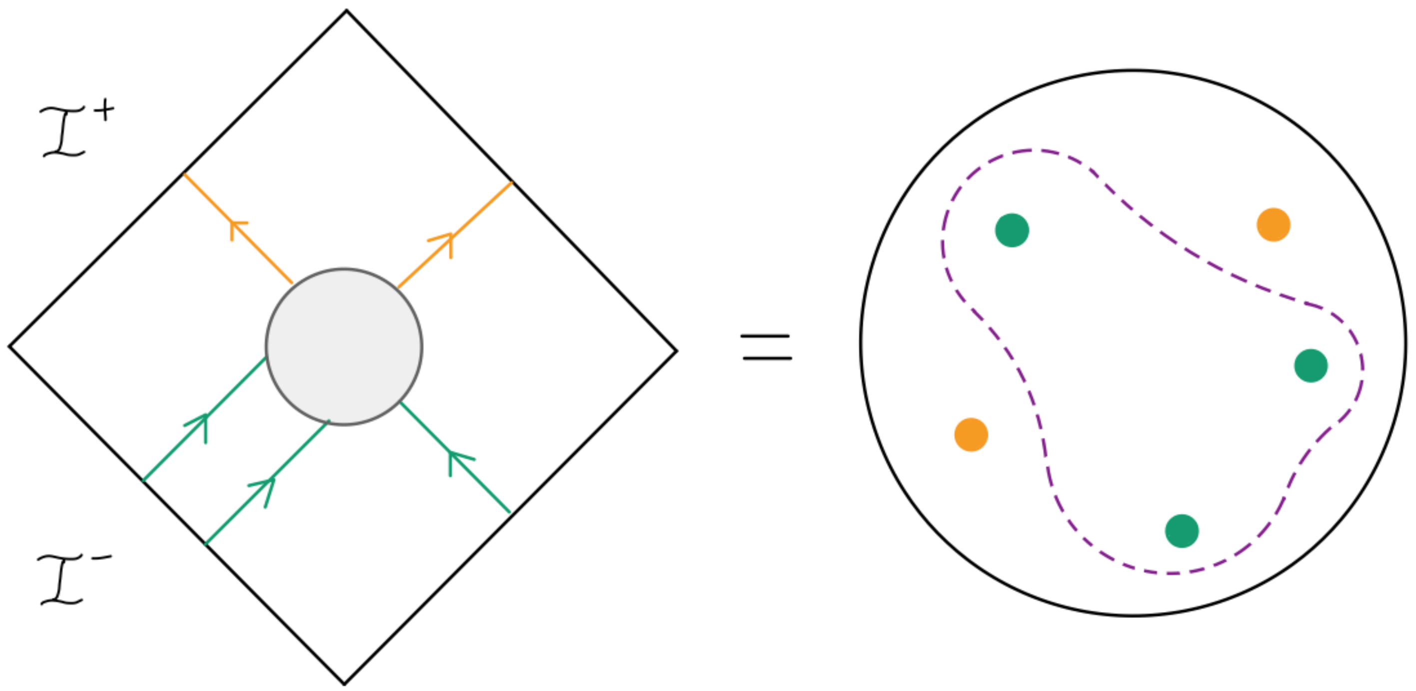
the superrotation Ward Id takes the form of a 2D stress tensor Ward Id.

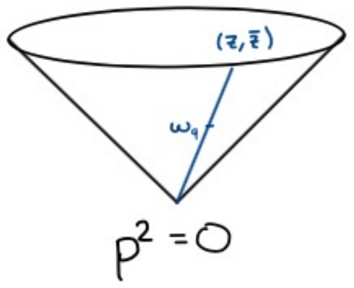
$$\langle T_{zz}\mathcal{O}_1 \cdots \mathcal{O}_n \rangle = \sum_{k=1}^{n} \left[ \frac{h_k}{(z-z_k)^2} + \frac{\Gamma^{z_k}_{z_k z_k}}{z-z_k} h_k + \frac{1}{z-z_k} \left( \partial_{z_k} - |s_k| \Omega_{z_k} \right) \right] \langle \mathcal{O}_1 \cdots \mathcal{O}_n \rangle$$

$$\underbrace{\text{subleading soft graviton theorem} = \text{Ward identity for 4D superrotations}}_{\text{the asymptotic symmetry is physical}} = \text{Ward identity for 2D stress tensor}$$

$$\text{subleading soft graviton theorem} = \underbrace{\text{Ward identity for 4D superrotations} = \text{Ward identity for 2D stress tensor}}_{\text{we should look for a 2D dual CFT}}$$

# 4D Soft Mode = 2D Current



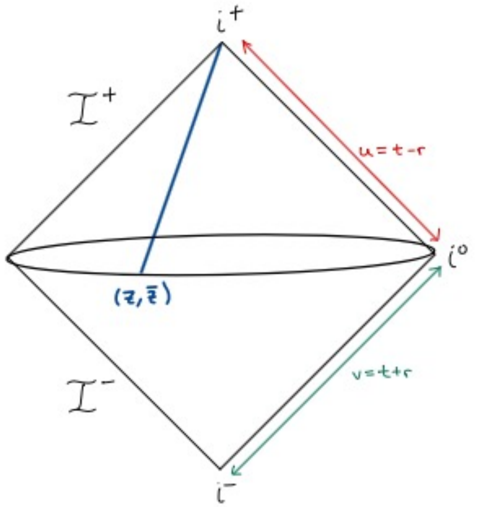



# 4D Amplitude = 4D Correlator

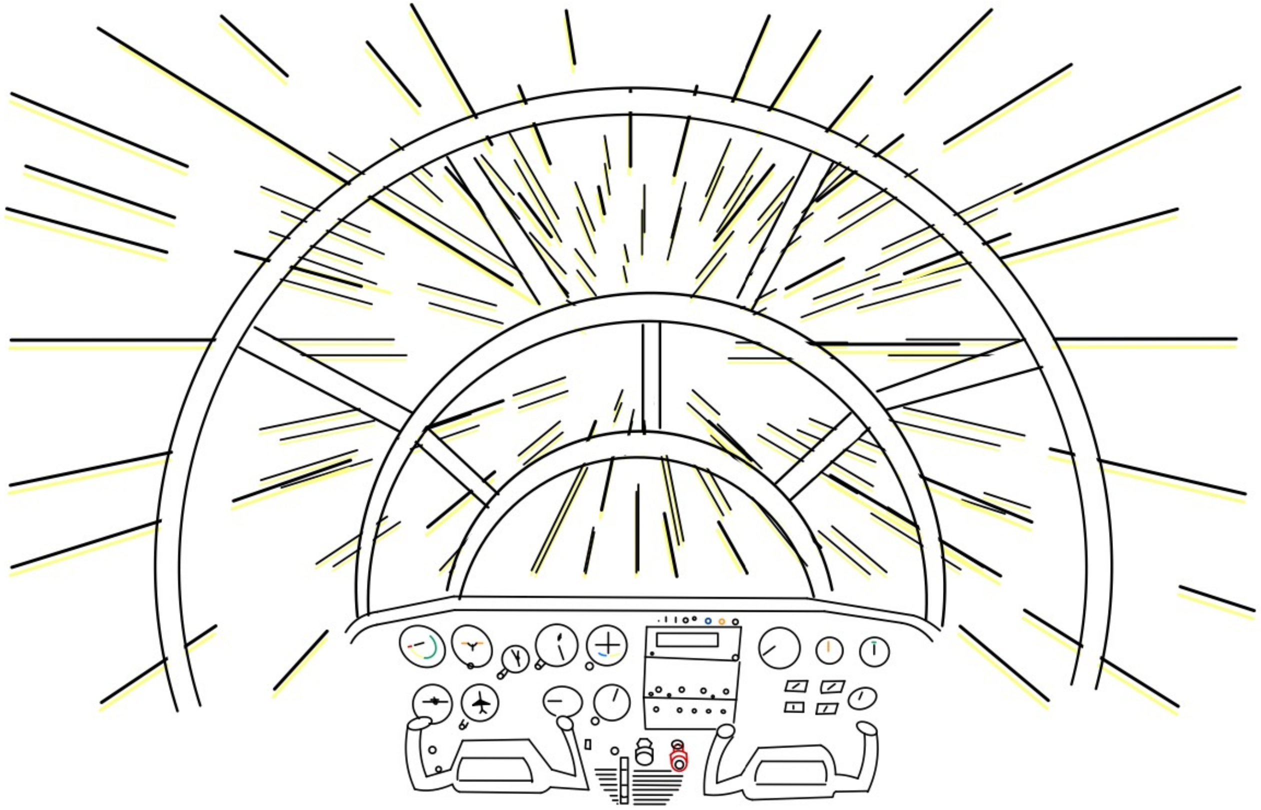
LSZ $\Leftrightarrow$ Extrapolate Dict.

$$\langle out|S|in\rangle_{boost} = \prod_i \lim_{r\to\infty} \int_{-\infty}^{\infty} \mathrm{d}\nu_i \, \nu_i^{-\Delta_i} \, \langle r\Phi(\nu_1, r, z_1, \bar{z}_1)...r\Phi(\nu_n, r, z_n, \bar{z}_n)\rangle$$

$$\nu = \{u, v\}$$



# 4D Amplitude = 2D Correlator

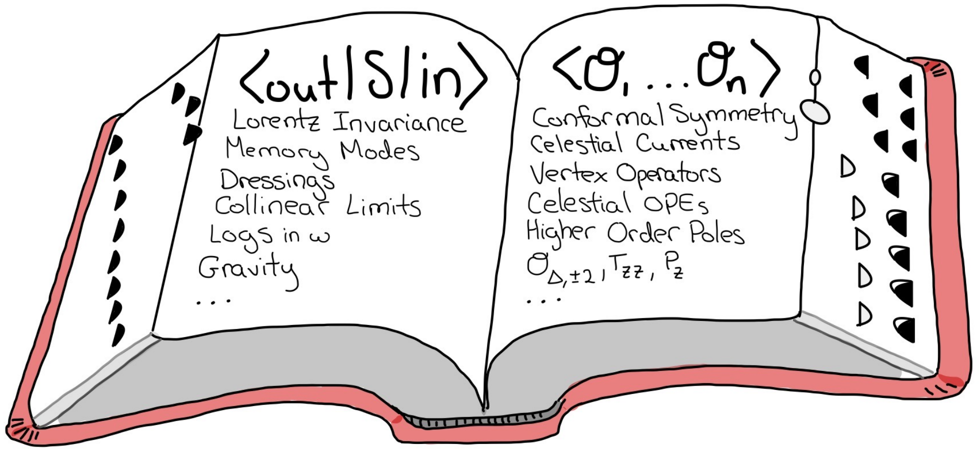


# 4D Amplitude = 2D Correlator

4D Lorentz invariance = 2D global conformal symmetry

$$\langle \mathcal{O}^{\pm}_{\Delta_1}(z_1,\bar{z}_1)\ldots\mathcal{O}^{\pm}_{\Delta_n}(z_n,\bar{z}_n)\rangle = \prod_{i=1}^{n}\int_0^{\infty} d\omega_i \omega_i^{\Delta_i-1}\langle out|\mathcal{S}|in\rangle$$

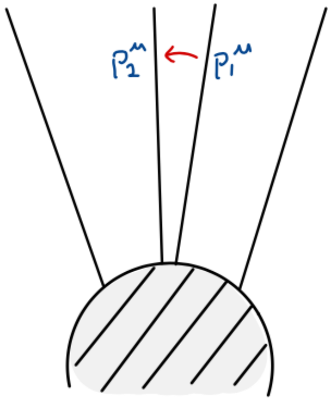
If we go to a boost basis, amplitudes transform as CFT correlators under the Lorentz group.

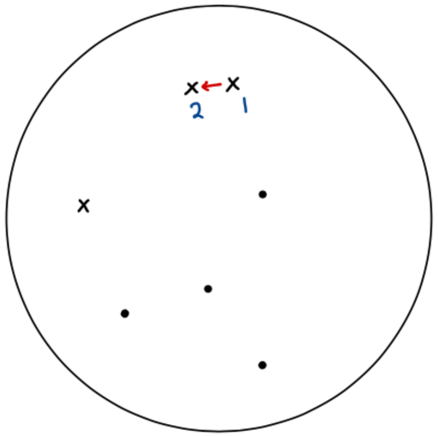


# Collinear Limit = OPE

$$\mathcal{O}_{\Delta_1,+2}(z_1,\bar z_1)\mathcal{O}_{\Delta_2,+2}(z_2,\bar z_2) \sim -\frac{\kappa}{2}\frac{\bar z_{12}}{z_{12}}B(\Delta_1-1,\Delta_2-1)\mathcal{O}_{\Delta_1+\Delta_2,+2}(z_2,\bar z_2)+\ldots\,,$$

$$\begin{aligned}\mathcal{O}_{\Delta_1,+2}(z_1,\bar z_1)\mathcal{O}_{\Delta_2,-2}(z_2,\bar z_2) \sim &-\frac{\kappa}{2}\frac{\bar z_{12}}{z_{12}}B(\Delta_1-1,\Delta_2+3)\mathcal{O}_{\Delta_1+\Delta_2,-2}(z_2,\bar z_2)\\ &-\frac{\kappa}{2}\frac{z_{12}}{\bar z_{12}}B(\Delta_1+3,\Delta_2-1)\mathcal{O}_{\Delta_1+\Delta_2,+2}(z_2,\bar z_2)+\ldots\,,\end{aligned}$$



# 2D Radial Quantization → More Symmetries

For special weights, the SL(2,C) multiplets have primary descendants.

$$H^k(z,\bar{z}) \;:=\; \lim_{\epsilon\to 0} \epsilon\, \mathcal{O}_{k+\epsilon,2}(z,\bar{z}), \quad \Delta = k \;=\; 2,1,0,-1,\ldots$$

Assuming these multiplets shorten, we have

$$H^k(z,\bar{z}) \;=\; \sum_{m=\frac{k-2}{2}}^{\frac{2-k}{2}} \bar{z}^{-\frac{k-2}{2}-m}\, H^k_m(z)\;, \qquad\qquad w_n^p \;=\; \frac{1}{\kappa}\,(p-n-1)!(p+n-1)!\, H_n^{-2p+4}$$

# 2D Radial Quantization → More Symmetries

Complexifying the celestial sphere variables and defining a holomorphic commutator

$$[A, B](z) \;=\; \frac{1}{2\pi i}\oint_z dw\, A(w)B(z)$$

gives a $Lw_{1+\infty}$ symmetry algebra for appropriately rescaled modes

$$\left[w_n^p, w_m^q\right](z) \;=\; \left[n(q-1) - m(p-1)\right] w_{m+n}^{p+q-2}(z)$$

# Celestial Algebra = Sym of SDG

conformally soft theorems

holomorphic collinear limits

$Lw_{1+\infty}$ in gravity, YM analog, …

**Holographic Symmetry Algebras**

gravitational phase space

twistor theory

twisted holography

# Celestial Algebra = Chiral Algebra

conformally soft theorems

holomorphic collinear limits

$Lw_{1+\infty}$ in gravity, YM analog, …

**Holographic Symmetry Algebras**

gravitational phase space

twistor theory

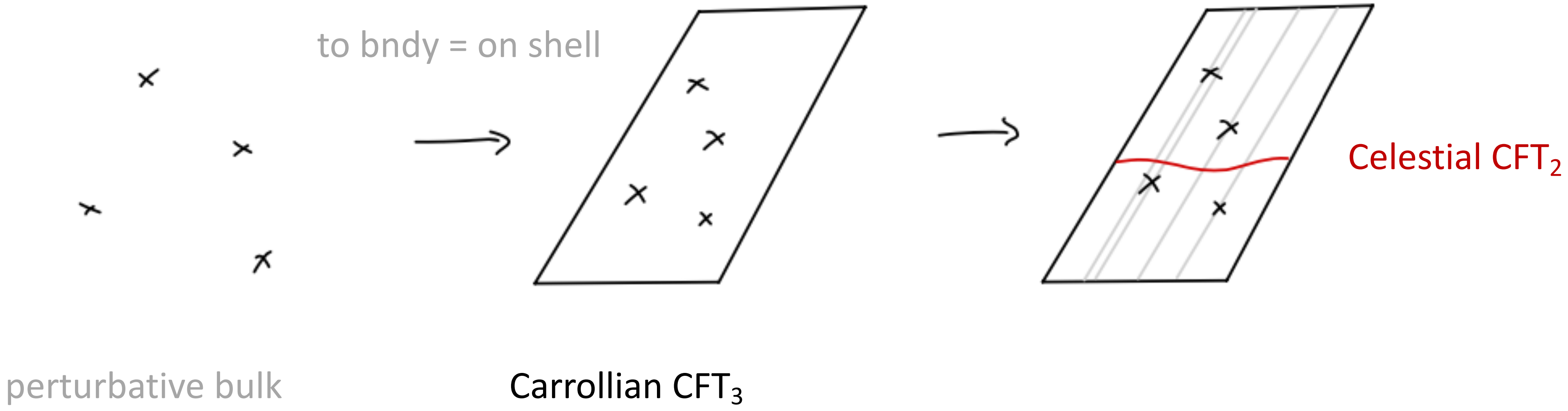
twisted holography

Do these symmetries beyond tree level, or the self-dual sector?

Can we realize them in the matter sector?

Can we really complexify the celestial sphere to define these currents?

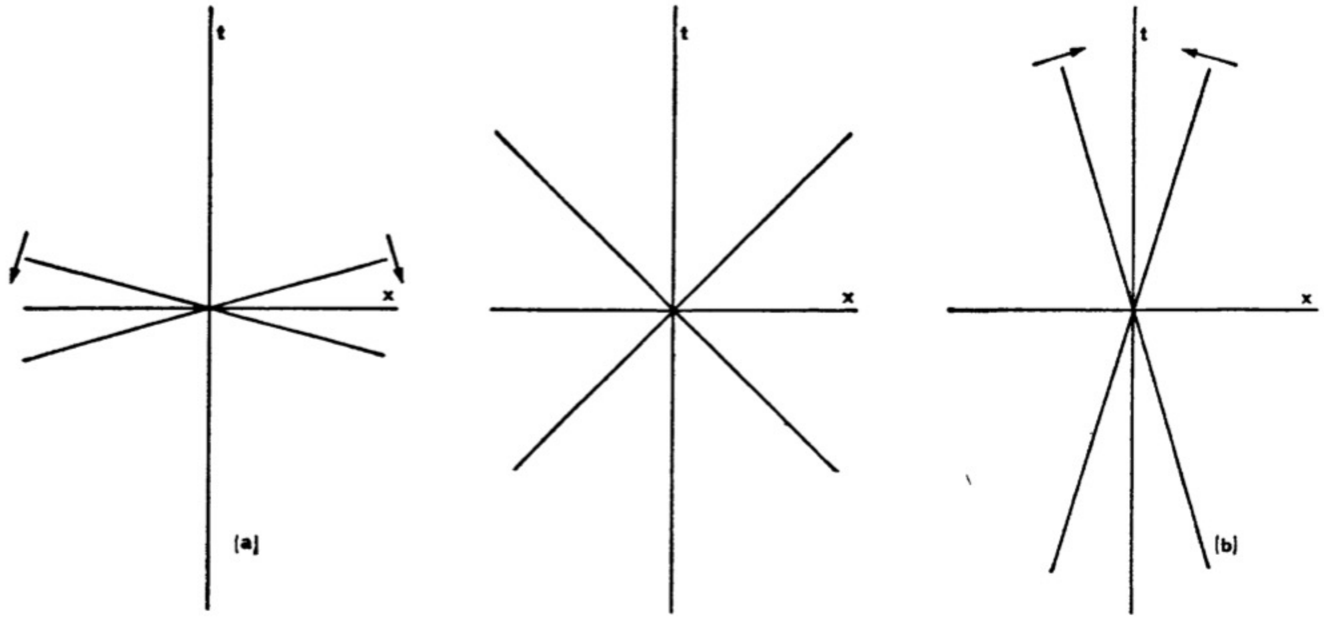
# Celestial = Carrollian



# Celestial = Carrollian

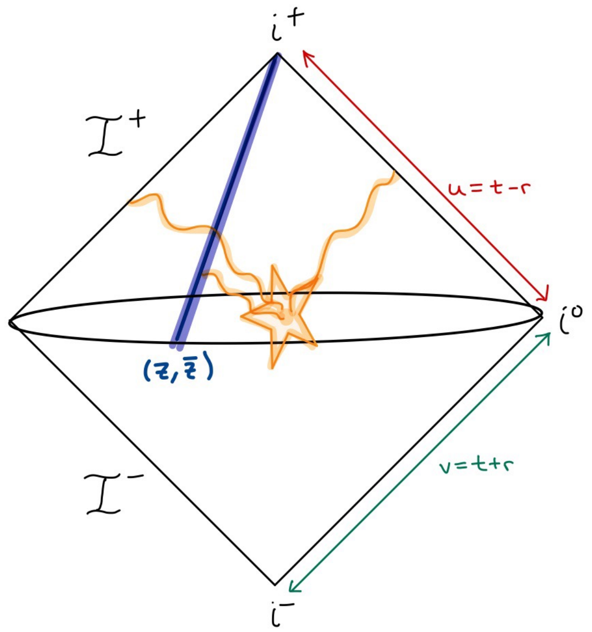
$$ds^2 = -c^2 dt^2 + d\vec{x}^2$$

| $c \to \infty$ | $c = 1$ | $c \to 0$ |
|---|---|---|
| Galilean | Lorentzian | Carrollian |



# Hard Charges = Light Ray Operators

$$Q^{\mathscr{I}^+}_{H,matter} \quad \Leftrightarrow \quad \mathcal{E}(z,\bar{z}) = \lim_{r\to\infty} r^2 \int du\, T_{uu}(u,r,z,\bar{z})$$



# Hard Charges = Light Ray Operators

**CCFT**

$$\langle \mathbb{O}^{\pm}_{\Delta_1}(z_1, \bar{z}_1)...\mathbb{O}^{\pm}_{\Delta_n}(z_n, \bar{z}_n)\rangle$$

Single Particle

Exclusive

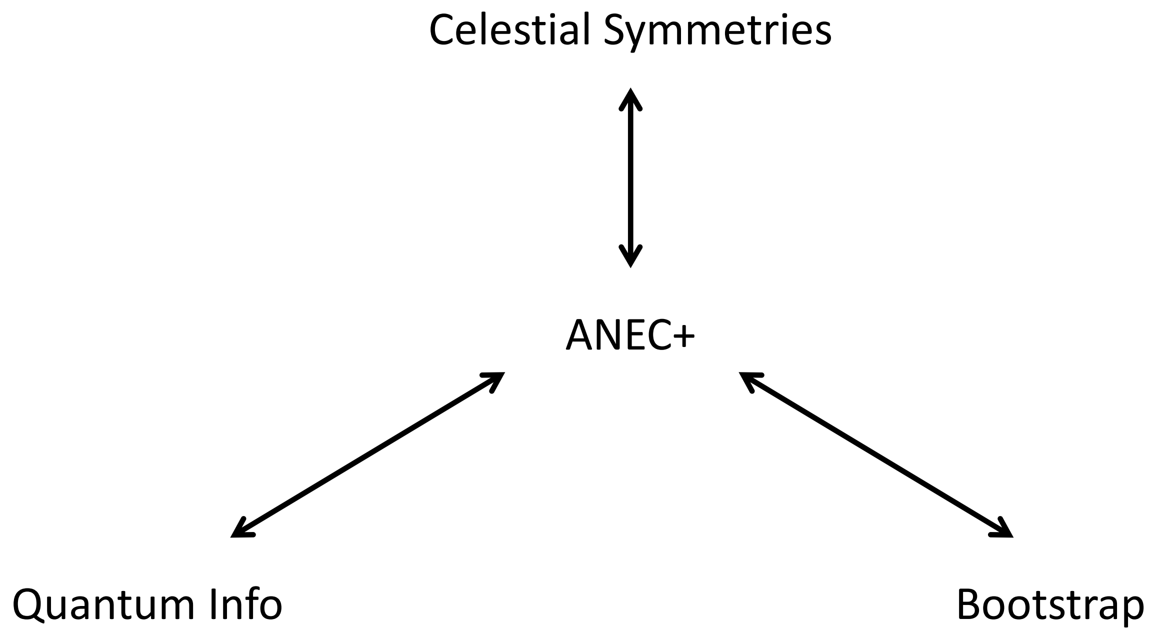
**vs**

**Conformal Colliders**

$$\frac{\langle 0|\mathcal{O}^\dagger \mathcal{E}(\theta_1)\cdots\mathcal{E}(\theta_n)\mathcal{O}|0\rangle}{\langle 0|\mathcal{O}^\dagger\mathcal{O}|0\rangle}$$

2-Particle

Inclusive



Celestial Symmetries
ANEC+
Quantum Info
Bootstrap

Amplitudes

soft and collinear factorization

IR finite S-matrix

It From Qubit

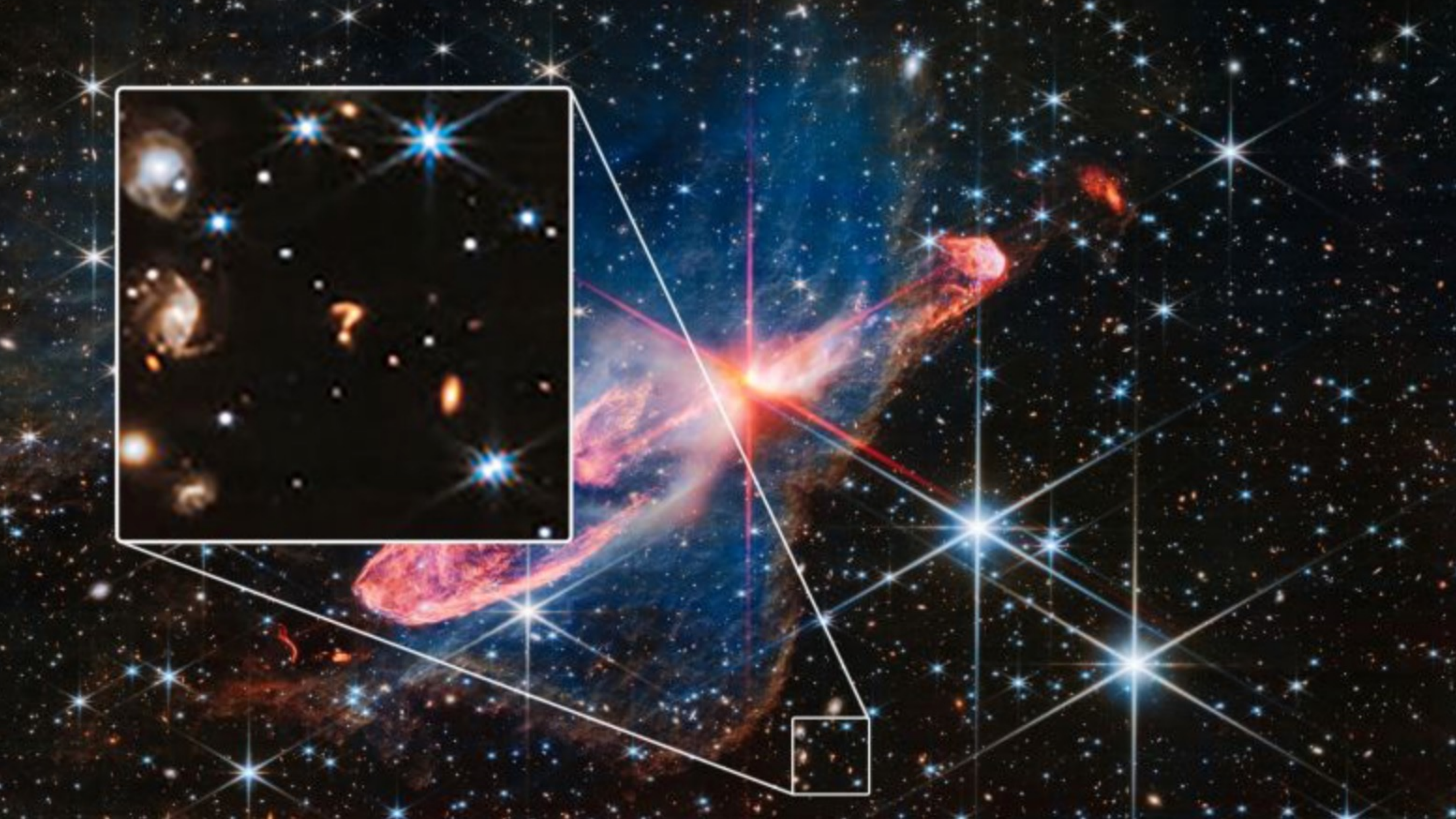
conformal colliders

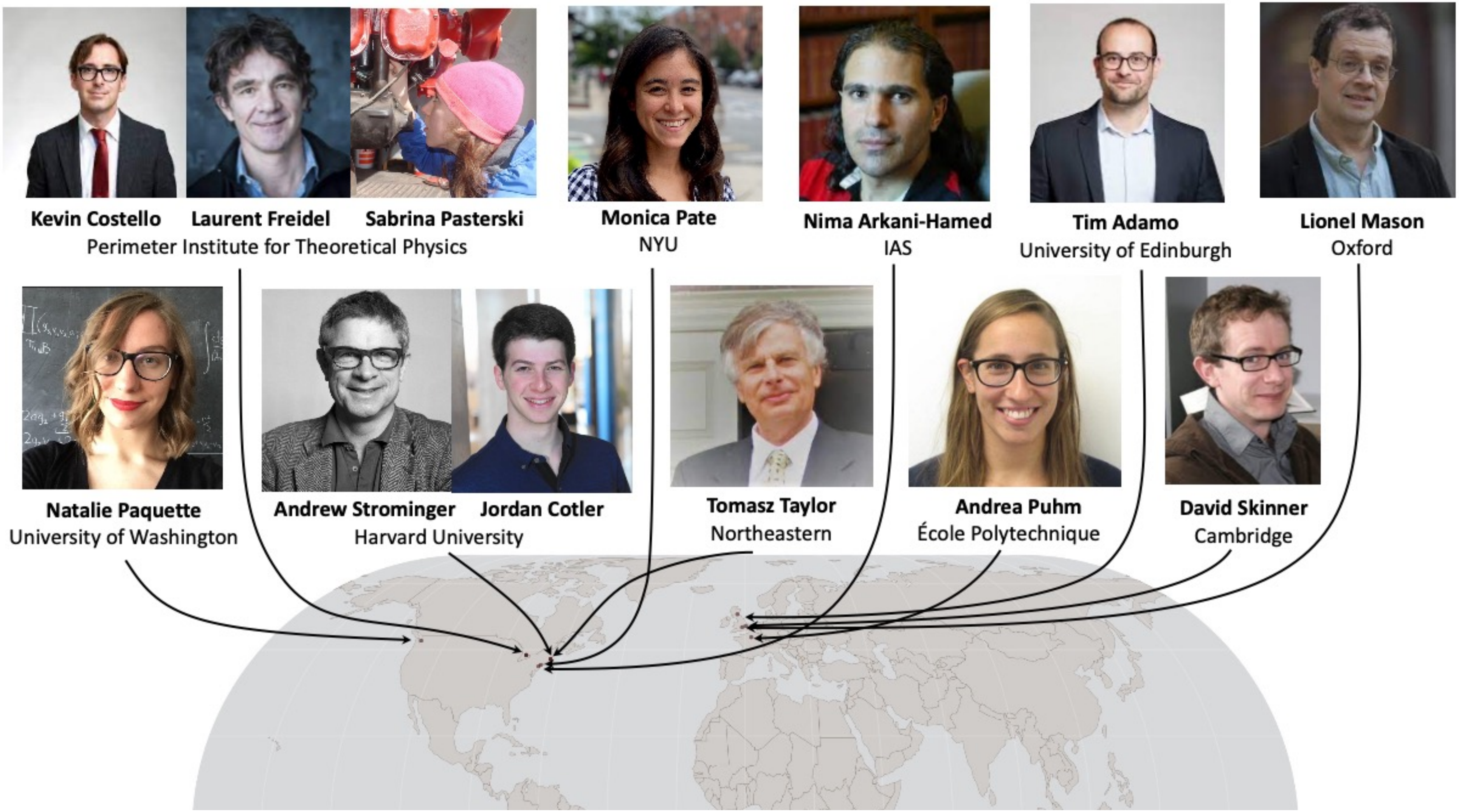
detector operators

light ray operators

complex spin

Bootstrap

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EFFECTIVE FIELD THEORY
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QED
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