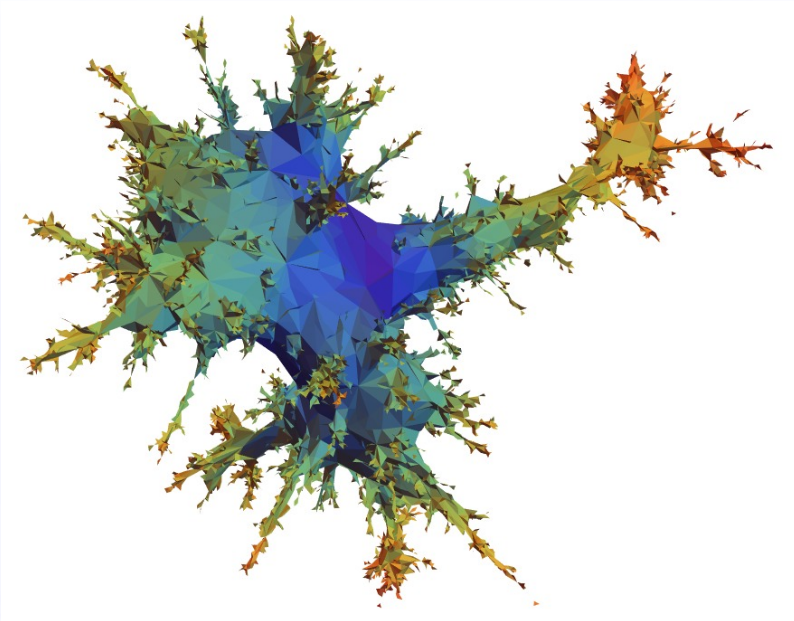
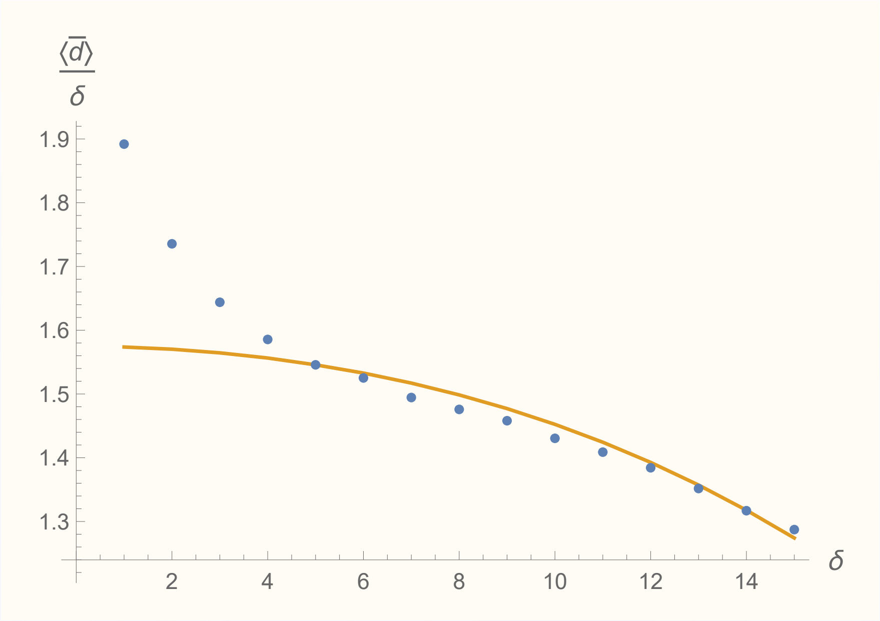


typical quantum spacetime (© T. Budd)



expectation value of quantum spacetime observable

CDT as computational gateway to nonperturbative Quantum Gravity

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Puzzles in the QG Landscape

PI, 24 Oct 2023

The two worlds of quantum gravity

nonperturbative QG, including Planckian "spacetime foam"

- large quantum fluctuations @ℓ_{Pl}
- no background, no effective $g_{\mu\nu}$
- optional: fundamental discrete-ness, extended objects, drop QFT tenets, "radical" ideas, …

QG must describe qu. spacetime, associated phenomenology

observables? uniqueness? how to do any computations?

classical limit?

effective QG/perturbative QG, nonperturbatively enhanced

- GR metric fields $g_{\mu\nu}(x)$
- (flat) background metric
- optional: gauge-gravity duality, nonperturbative FRG flow equations, modified GR …

QG must resolve singularities, black-hole information loss, …

observables? quantum effects tiny? range of applicability?

nonperturbative quantum limit?

Where should quantum gravity be going?

- **P**erturbative vs. **N**on**P**erturbative QG — different ingredients, problems and expectations: complementary or contradictory?
- most QG research today is in **P**-world, using covariant QFT methods
- our (classically trained) gravity "intuition" works well in **P**-world; extrapolating it naïvely to **NP**-world, we get lots of "no-go folklore"
- still, we hope for *large* QG effects, presumably from **NP**-world

Q: Are we stuck then?

A: No, but we may need a change of perspective.

Q: Where will new insights for QG come from?

A: Not from *physical* experiment or observation, but from *numerical* experiments, namely ***lattice quantum gravity***.

What's the role of Causal Dynamical Triangulations?

- CDT is not a QG *approach*, but a *method*, in the sense of a valid *effective* description of QG near ℓ_{Pl}, independent of the UV completion
- it reflects our best understanding of lattice quantum gravity today; *CDT is to gravity what lattice QCD is to nonabelian gauge field theory*
- it took a long time to address the obstacles to realizing this analogy: dynamical and causal nature of spacetime, unboundedness of the action, Wick rotation, manifest gauge (diffeomorphism) invariance, …
- theoretical & numerical fundamentals in 4D well tested since 2004; selected observables have sensible UV behaviour and classical limit

Q: So has CDT solved QG? Are we done?

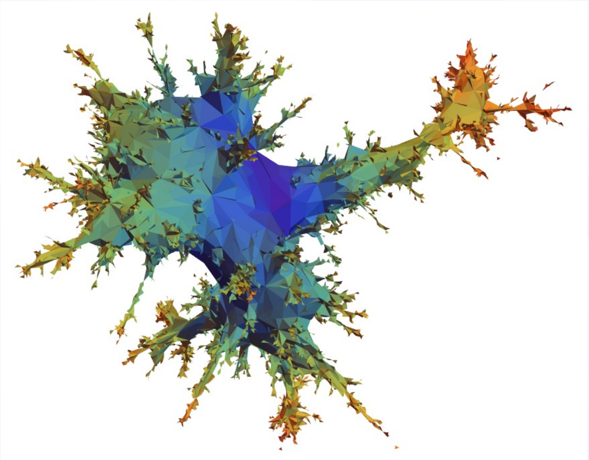
A: No, but we have a unique observational window at $\backsim$ 4–20 ℓ_{Pl} .

Q: What's the big deal?

A: A functioning computational tool and "experimental" data!

What new perspectives does it offer?

- a focus on physics, rather than technical issues of the formalism



- we can understand the nature of NP QG from *data* = measurements of <u>quantum observables</u>
- work with a concrete realization of "spacetime foam" (∽ nowhere differentiable geometries)
- nonlocal character of observables reflects *features* of this Planckian regime, not technical inconveniences
- throws new light on what it may mean to "solve" quantum gravity, considering the available NP theoretical/calculational tools: rather than being "just numerics", lattice QG may be the ***primary gateway*** to understanding the strongly coupled quantum regime @ℓ_{Pl}
- *new* mathematics: random geometry, beyond-Riemannian geometry
- best bet regarding phenomenology: early universe cosmology

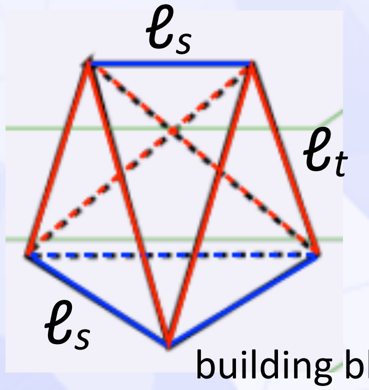
Causal Dynamical Triangulations: the basics

$$Z=\int_{g\in\frac{Lor(M)}{Diff(M)}}\mathcal{D}g\;\mathrm{e}^{iS_{\rm grav}[g]}$$

- gravitational path integral over metric d.o.f., *nonperturbative* (NP), background-independent, Lorentzian signature, 4D, not "grand-unified"
- building on Euclidean "dynamical triangulations", define a new NP Lorentzian 2D path integral: *exactly soluble ⇒ signature matters!*

J. Ambjørn, R.L., NPB 536 (1998) 407

- CDT combines emphasis on geometry with path integral covariance (no split $g_{\mu\nu}=\eta_{\mu\nu}+h_{\mu\nu}$, no 3 + 1 decomposition)
- uses a regularized version of the space of geometries, $\mathcal{G}(M)=Lor(M)/Diff(M)$: piecewise flat, simplicial manifolds T



building block of 4D CDT

- minimal GR ingredients **+** standard Q(F)T methods, adapted to dynamical geometry **+** numerical methods **=** **new territory near ℓ_{Pl}**
- 2D random geometry is a hot topic in maths!

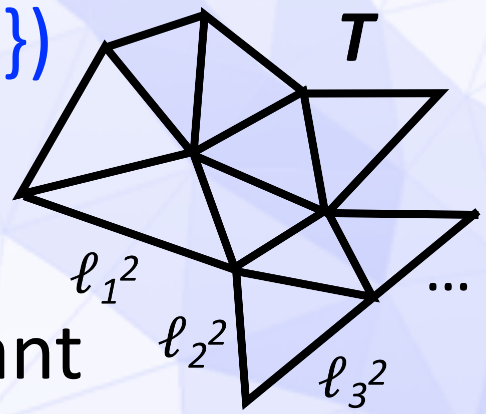
T. Budd, arXiv:2212.03031

Putting quantum gravity on a lattice, correctly

General strategy: lattice acts as a regulator, with UV cutoff a; search for a continuum limit by approaching a second-order phase transition in the limit $a \to 0$ while renormalizing bare couplings appropriately; attain *universality* (independence of regularization); this is **not** "discrete QG"

- "reaches where other methods don't", subject to numerical limitations; if it exists, continuum theory is essentially *unique*
- "QCD-like" lattice QG (≥ 1979): put some first-order formulation of GR (tetrad $e_\mu{}^A$ + spin connection $\omega_\mu{}^{AB}$) on a fixed hypercubic lattice; problem: diffeomorphism symmetry badly broken; no interesting results
- *geometric* lattice QG (≥ 1981): based on *approximation* "GR without coordinates" $(M, g_{\mu\nu}(x)) \to (T, \{\ell_i{}^2\})$, $S_{grav}[g] \to S^{Regge}(T, \{\ell_i{}^2\})$

T. Regge, Nuovo Cim. A19 (1961) 558



- diffeomorphism-invariance manifest, work directly on $\mathcal{G}(M)$; CDT ($\ell^2 = \pm a^2$) implementation is labelling-invariant

The path integral (PI) according to CDT

$$Z = \int_{\mathcal{G}(M)} \mathcal{D}g \, \mathrm{e}^{iS_{\mathrm{grav}}[g]} \to Z^{\mathrm{CDT}} = \lim_{a \to 0} \sum_{\substack{inequiv.\\ causal\\ triang.\,T}} \frac{1}{C(T)} \mathrm{e}^{iS^{\mathrm{Regge}}[T]}$$

bare action

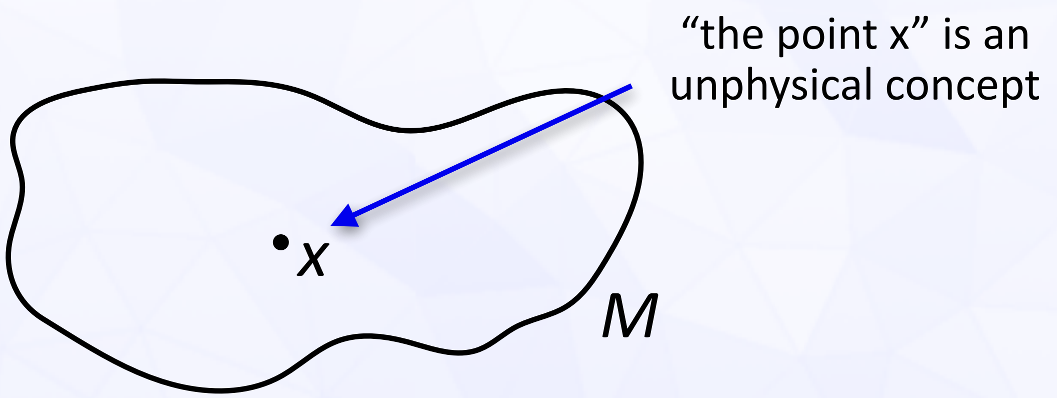
discrete symmetries of T

- usually, can't evaluate complex PI, do Euclidean *∫Dg exp(-S^eu)* <u>instead</u>
  - ☑ CDT has a well-defined analytic continuation ("Wick-rotation")
- usually, hard to renormalize compatible with diffeomorphism symmetry
  - ☑ CDT has no residual symmetries, has a geometric cutoff *a*
- usually, PI highly divergent, no unique renormalization;
  - ☑ numerical evidence of exponential bound on # of configurations
- usually, cannot do any computations, PI not Gaussian
  - ☑ CDT amenable to Monte Carlo simulation; get quantitative results
- usual problem: why should PI lead to a unitary theory?
  - ☑ CDT reflection-positive w.r.t. discrete "proper time"

CDT quantum gravity: results

- physics of ***quantum spacetime*** is captured by invariant ***quantum observables*** $\hat{\mathcal{O}}$:

$$\langle\hat{\mathcal{O}}\rangle = \frac{1}{Z}\int \mathcal{D}g\, \mathcal{O}[g]\, \mathrm{e}^{-S_{\mathrm{grav}}[g]}$$



- observables in gravity are nonlocal integrals of scalars, like $\int_M d^4x\sqrt{g}\,R(x)$
- this also reflects the absence of meaningful reference frames @ℓ_{Pl}
- "expectation management": your favourite **P**-world QG question will probably ***not*** a have Planckian implementation (this is a feature)
- quantum gravity signature: CDT predicts a reduction 4 → 2 of the ***spectral dimension*** @ℓ_{Pl}, which is reproduced across approaches, and a conjectured *universal* property of QG

J. Ambjørn, J. Jurkiewicz, R.L., PRL 95 (2005) 171301

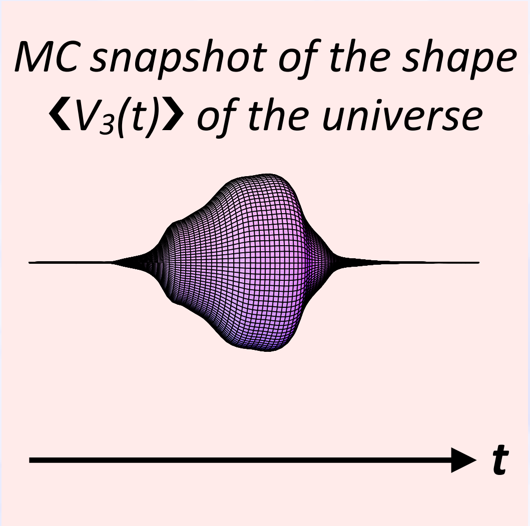
S. Carlip, CQG 34 (2017) 193001

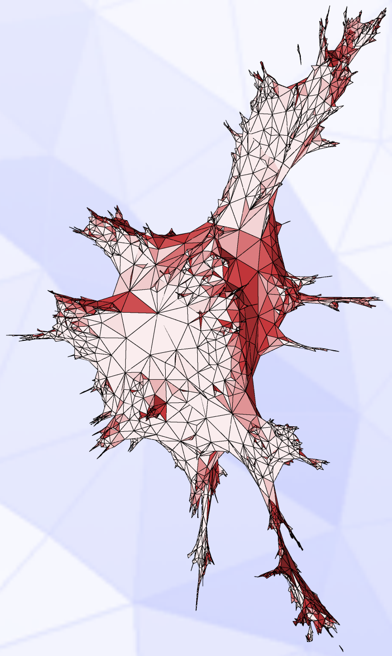
Key result: emergence of classicality from CDT

For $M = I \times S^3$, the measured average shape $\langle V_3(t) \rangle$ (spatial volume as a function of proper time) of the dynamically generated quantum spacetime in CDT matches that of a classical de Sitter space.

J. Ambjørn, A. Görlich, J. Jurkiewicz, R.L., PRL 100 (2008) 091304, PRD 78 (2008) 063544

Remarkable, but from the global conformal mode alone we cannot conclude this ***is*** (Euclidean) de Sitter space ($\not\approx$ symmetry-reduced quantum cosmology).

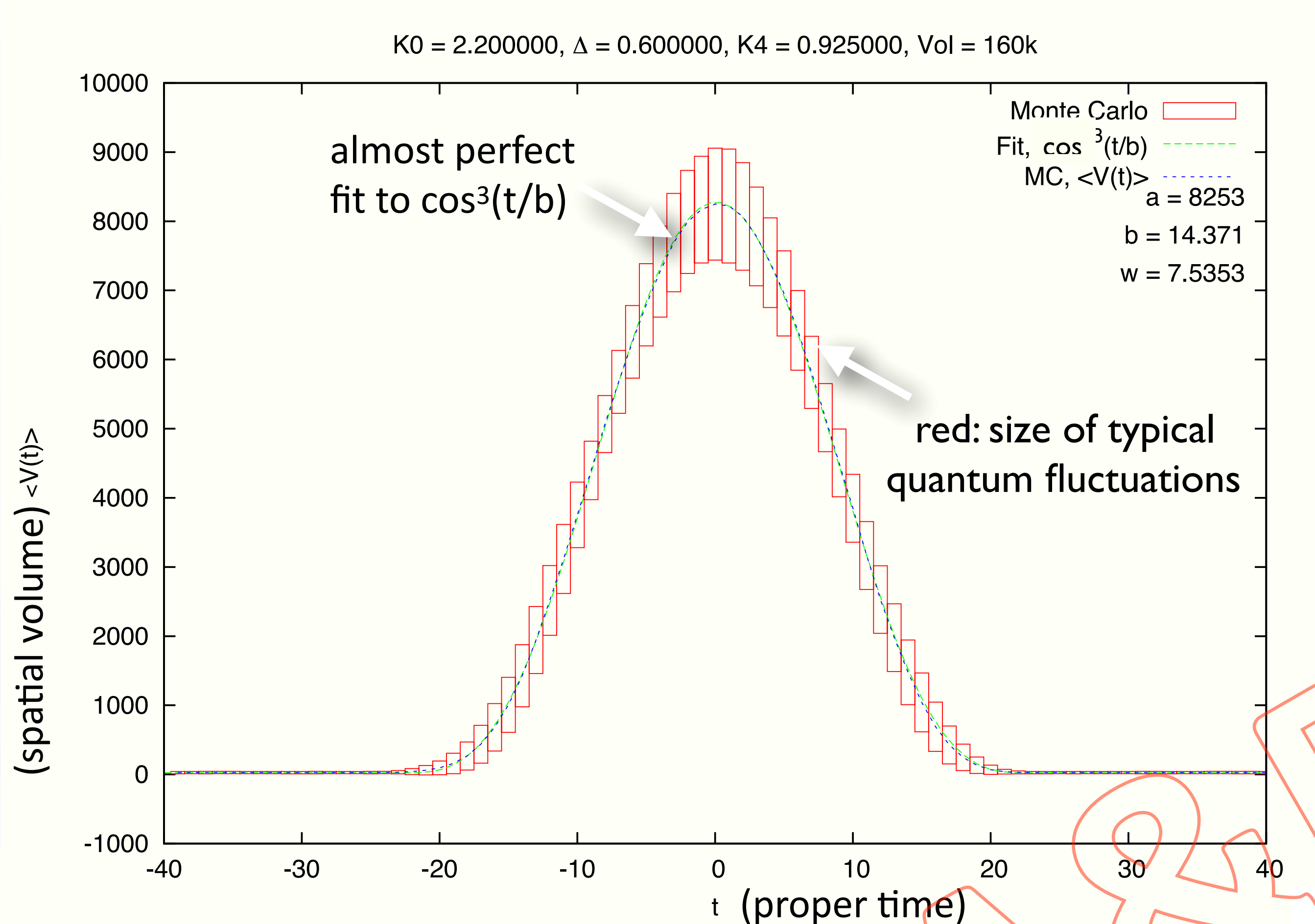




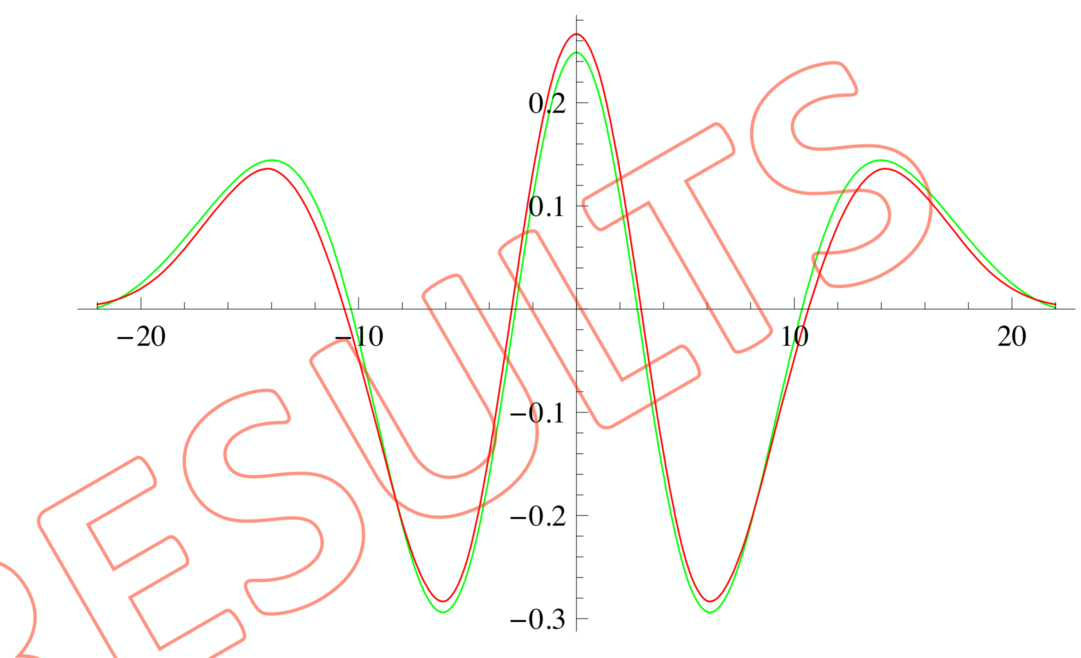
However, we have recently managed to construct a notion of ***quantum Ricci curvature*** that is well defined in a Planck regime. It yields a finite average curvature compatible with de Sitter space!

N. Klitgaard & RL, Eur. Phys. J. C80 (2020) 990

The universe is de Sitter-shaped

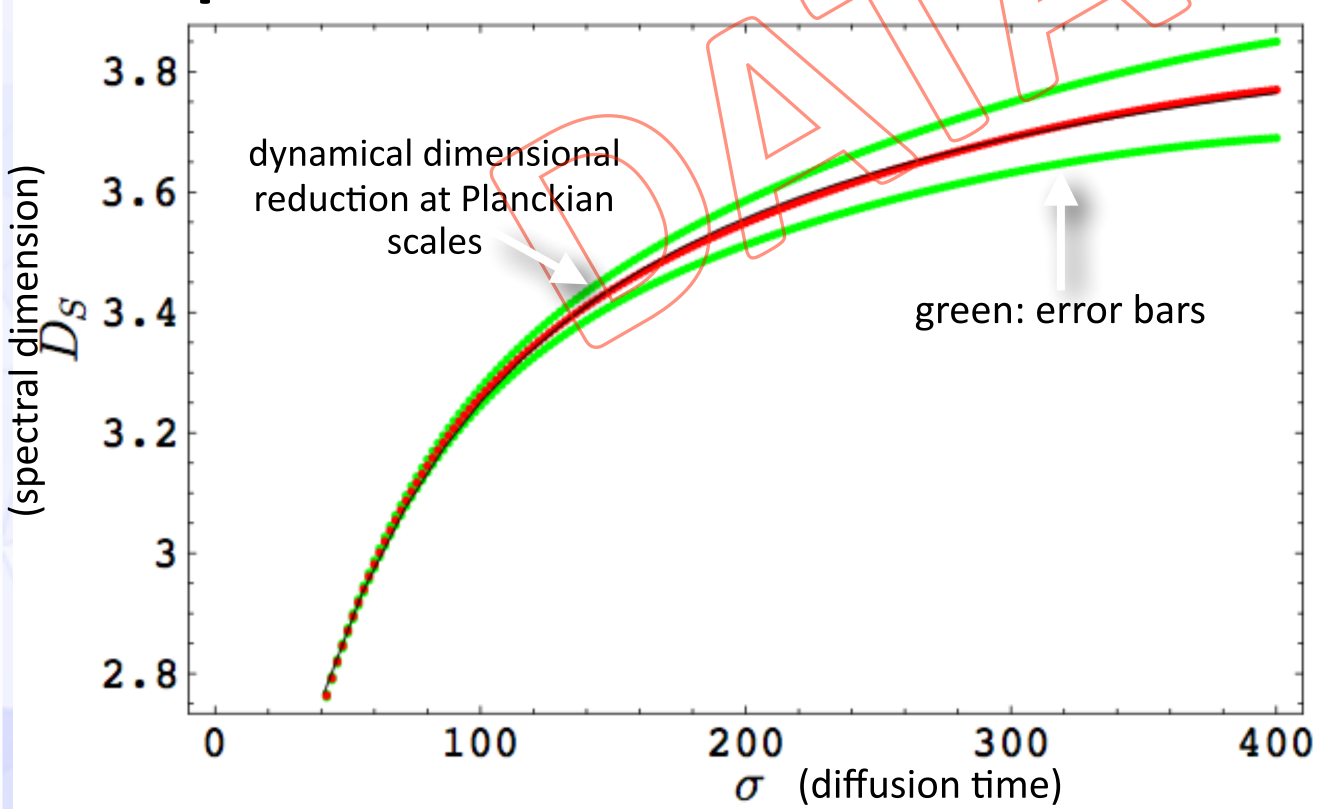


Volume fluctuations around de Sitter

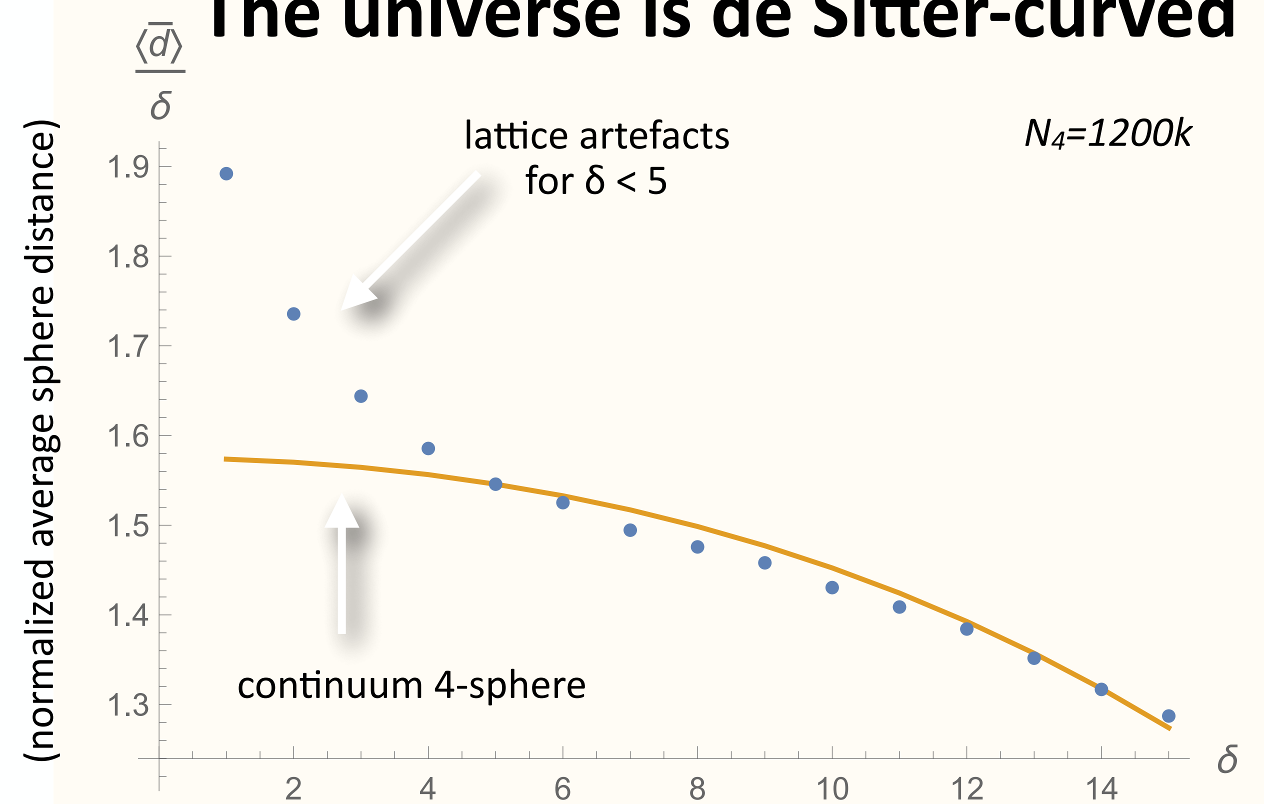


(low-lying eigenmode matches with semiclassical expectation)

Spectral dimension of the universe



The universe is de Sitter-curved



(quantum Ricci curvature of the dynamically generated dS universe)

Relation to our actual universe

CDT predicts a universe with ***$\Lambda > 0$***, which is 4D on large scales, and whose ***shape and average curvature*** are compatible with those of a ***dS space***, matching our current understanding of the early universe.

These properties have been obtained *from first principles*, from data measured in our small observational window; we have also reverse-engineered the effective action for the scale factor *$a(t) \backsim V_3(t)^{1/3}$*.

At what scales and how does gravity interact with ***matter***?

$$Z = \int_{\mathcal{G}(M)} \mathcal{D}g \int_{\Phi} \mathcal{D}\phi \; \mathrm{e}^{i(S_{\mathrm{grav}}[g]+S_{\mathrm{matter}}[g,\phi])}$$

Investigations of CDT coupled to matter fields have so far not found a significant impact on the geometry ⇒ “matter doesn’t matter @ℓ_{Pl}”?

The way ahead: the 'real' quantum universe

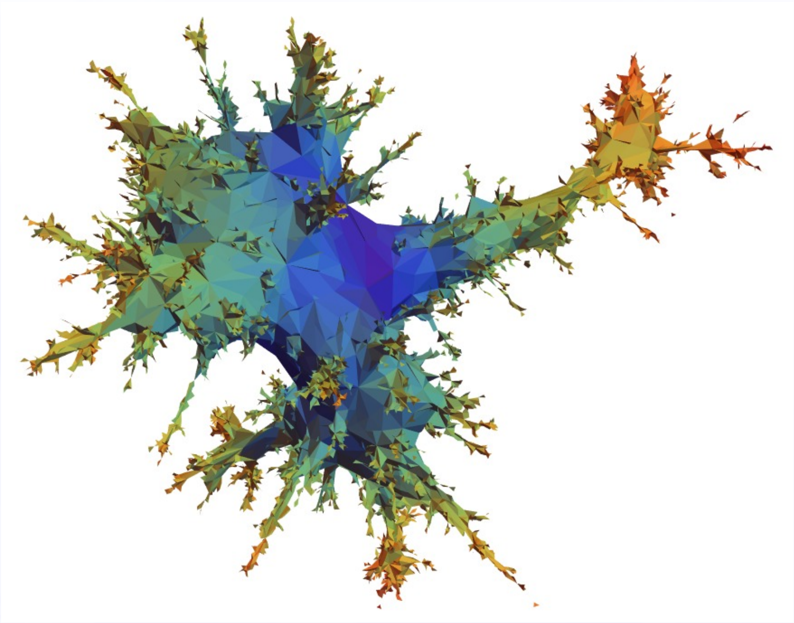
The new ***quantum curvature*** allows us to construct ***new, interesting quantum observables***, to quantify other physical properties of the emergent quantum "de Sitter" universe and relate them to standard models of early-universe physics. **R.L., arXiv:2306.13782**

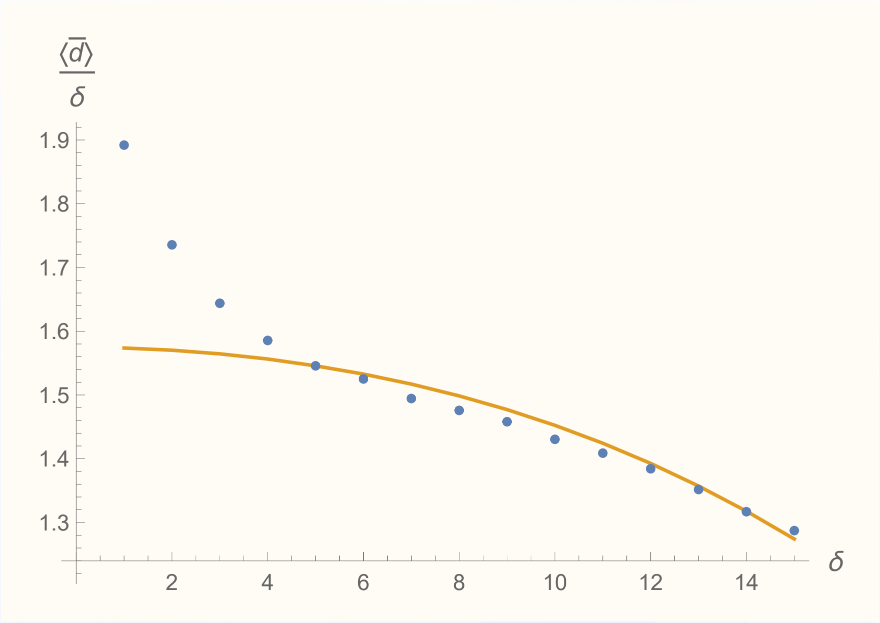
- nonperturbative quantum measures of homogeneity and isotropy; prototype construction in 2D **A. Silva, R.L., PRD 107 (2023) 10256**
- diffeomorphism-invariant two-point functions (spacetime and spatial); curvature correlator in 2D QG **J. van der Duin, R.L., to appear**

4D implementations are nontrivial; no guarantee of useful results!

If *you* want to become an experimental quantum gravity researcher, check our new CDT simulation guide! **J. Brunekreef, A. Görlich, R.L., arXiv:2310.16744**

<u>CDT reviews</u>: J. Ambjørn, A. Görlich, J. Jurkiewicz, R.L., Phys. Rep. 519 (2012) 127, arXiv: 1203.3591; R.L., Class. Quant. Grav. 37 (2020) 013002, arXiv:1905.08669

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Thank you!