# The Case for Renormalizable Quantum Gravity: from local to nonlocal approaches (and back!)


**Luca Buoninfante**

NORDITA
The Nordic Institute for Theoretical Physics


*Puzzles in the Quantum Gravity Landscape: viewpoints from different approaches*

**Perimeter Institute, 24th October 2023**

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$D = 1 + 3$

$(- \ + \ + +)$

$c = 1 = \hbar$

$\Lambda \geq 0$

# Motivations

Einstein's General Relativity: $$S_{EH} = \frac{M_p^2}{2} \int d^4x \sqrt{-g}(R - 2\Lambda)$$

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'Unique' (strictly) renormalizable QFT of gravity in $D = 4$:

[Stelle PRD (1977)]

$$S = \frac{1}{2}\int d^4x\,\sqrt{-g}\left(M_p^2(R - 2\Lambda) + \frac{\alpha}{6}R^2 - \frac{\beta}{2}C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}\right)$$

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Massive spin-0: $m_0^2 = \frac{M_p^2}{\alpha}$,

$\alpha \sim 10^{10}$: natural explanation for inflation!

[Starobinsky, 1980+]

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[Starobinsky, 1980+]

Massive spin-2 ghost:

$$m_2^2 = \frac{M_p^2}{\beta} + \frac{2}{3}\Lambda\left(2\frac{\alpha}{\beta} + 1\right)$$

# Ghost instability: classical level

Ostrogradsky's theorem

*If a non-degenerate Lagrangian,* $L = L(x, \dot{x}, \ddot{x}, \ldots, x^{(n)})$*, depends on the n-th derivative of a single configuration variable* $x$*, with* $1 < n < \infty$*, then the Hamiltonian is unbounded from below.*

[Ostrogradsky (1850); Woodard (2015)]

# Ghost instability: classical level

## Runaway solutions

Example:

$$a\,\frac{d^4y(t)}{dt^4} + b\,\frac{d^2y(t)}{dt^2} + c\,y(t) = 0$$

$$\Longrightarrow \quad y(t) = C_1e^{-k_-t} + C_2e^{k_-t} + C_3e^{-k_+t} + C_4e^{k_+t}$$

$$k_\pm \equiv \frac{1}{\sqrt{2a}}\sqrt{-b \pm \sqrt{b^2 - 4ac}}$$

# Ghost instability: quantum level

Scalar toy model:

ghost

$$\mathcal{L} = \frac{1}{2}\phi(\Box - m^2)\left(1 - \frac{\Box}{M^2}\right)\phi - V(\phi) \implies \Pi(p) = \frac{1}{p^2 + m^2 - i\epsilon} - \frac{1}{p^2 + M^2 - i\varepsilon}$$

($\epsilon, \varepsilon > 0$ Feynman prescription)

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Optical theorem:

$$S^+ S = 1, \qquad S = 1 + iT,$$

$$1 = \sum_{\{n\}} c_n \, |n\rangle\langle n|, \qquad c_n > 0$$

$$\implies \quad 2Im\{\langle a|T|a\rangle\} = \sum_{\{n\}} c_n |\langle n|T|a\rangle|^2 \geq 0$$

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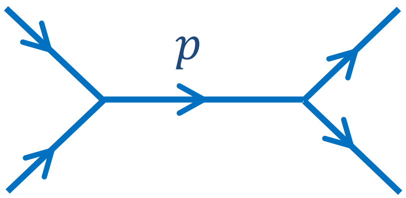
Tree-level example ($V \sim \phi^3$):




$$Im\{\langle a|T|a\rangle\} \sim \theta(p^0)[\delta(p^2 + m^2) - \delta(p^2 + M^2)]$$

It can be negative: violation of unitarity!

# Quatum Gravity Puzzle

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I consider two types of approaches:

1. Correspondence Principle applies:

2. Correspondence Principle does not apply:

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2. Correspondence Principle does not apply:

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1. Correspondence Principle applies: ghost must be classically harmless

(give up locality)

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$$+RF_R(-\Box)R+C_{\mu\nu\rho\sigma}F_C(-\Box)C^{\mu\nu\rho\sigma}+\cdots\Big], \qquad \Box = g^{\mu\nu}\nabla_\mu\nabla_\nu$$

2. Correspondence Principle does not apply:

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$$\left. + RF_R(-\Box)R + C_{\mu\nu\rho\sigma}F_C(-\Box)C^{\mu\nu\rho\sigma} + \cdots\right], \qquad \Box = g^{\mu\nu}\nabla_\mu\nabla_\nu$$

2. Correspondence Principle does not apply:

- Keep locality
- quantum features needed to make the ghost harmless
- (consistent) classical limit taken from the quantum theory

# Outline

- **Approach 1:** Correspondence Principle applies (give up ‘‘locality’’)
- **Approach 2:** Correspondence Principle does <u>not</u> apply
- Discussion

# Approach 1: beyond 4 derivatives

Scalar 4-derivative model:

$$\mathcal{L} = \frac{1}{2}\phi\Box\left(1 - \frac{\Box}{M^2}\right)\phi - V(\phi) \quad \Longrightarrow \quad \Pi(p^2) = \frac{1}{p^2} - \frac{1}{p^2 + M^2}$$

Generalized higher-derivative model:

$$\mathcal{L} = \frac{1}{2}\phi f(-\Box)\Box\phi - V(\phi) \quad \Longrightarrow \quad \Pi(p) = \frac{1}{f(p^2)p^2}$$

Is there any non-trivial $f(-\Box)$ such that the propagator is ghost-free?

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Is there any non-trivial $f(-\Box)$ such that the propagator is ghost-free?

YES! Nonlocality (infinite-order derivatives) can help!

# Approach 1: beyond 4 derivatives

## Local vs Nonlocal

Local Lagrangian:

$$\mathcal{L}_L \equiv \mathcal{L}_L(\phi, \partial\phi, \partial^2\phi, \dots, \partial^n\phi)$$

Nonlocal Lagrangian:

$$\mathcal{L}_{NL} \equiv \mathcal{L}_{NL}\left(\phi, \partial\phi, \partial^2\phi, \dots, \partial^n\phi, \dots, \log(\Box)\,\phi, e^{\Box}\phi, \frac{1}{\Box}\phi, \dots\right)$$

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Typical in standard perturbative QFT

# Approach 1: beyond 4 derivatives

## Local vs Nonlocal

Local Lagrangian:

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non-polynomially boundedness

# Approach 1: beyond 4 derivatives

Bare scalar field Lagrangian:

$$\mathcal{L} = -\frac{1}{2}\phi F(-\Box)\phi - V(\phi)$$

Entire function
(good IR limit: $F(-\Box) \to -\Box + m^2$)

Weierstrass' theorem:

$$F(-\Box) = e^{-\gamma(-\Box)} \prod_{i=1}^{N} (-\Box + m_i^2)^{r_i}, \qquad N \leq \infty,$$

$\gamma(-\Box)$ is an entire function

$N$ is the number of zeroes $m_i^2$; $r_i$ is the multiplicity of each zero

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Propagator:

$$\Pi(p^2) = \frac{e^{\gamma(p^2)}}{p^2 + m^2} \prod_{i=2}^{N} \frac{1}{\left(p^2 + m_i^2\right)^{r_i}}$$

# Approach 1: beyond 4 derivatives

$$F(-\Box) = e^{-\gamma(-\Box)} \prod_{i=1}^{N} (-\Box + m_i^2)^{r_i}, \qquad N \leq \infty,$$

- $N = 1,\ r_i = 1,\ \gamma(-\Box) = 0 \implies$ 2-derivative theory (Klein-Gordon)

$$F(-\Box) = -\Box + m^2$$

- $N = 2,\ r_i = 1,\ \gamma(-\Box) = 0 \implies$ 4-derivative theory with ghost

$$F(-\Box) = (-\Box + m^2)\left(1 - \frac{\Box}{M^2}\right)$$

- $\infty > N \geq 2$ and/or $r_i \geq 2$ (with $m_i$ real) $\implies$ ghosts!

# Approach 1: beyond 4 derivatives

$$F(-\Box) = e^{-\gamma(-\Box)} \prod_{i=1}^{N} \left(-\Box + m_i^2\right)^{r_i}, \qquad N \leq \infty,$$

- $N = 1,\ r_i = 1,\ \gamma(-\Box) \neq 0$

$\Longrightarrow$ infinite-order derivatives and one real zero:

$$F(-\Box) = e^{-\gamma(-\Box)}(-\Box + m^2)$$

Propagator:

$$\Pi(p^2) = \frac{e^{\gamma(-p^2)}}{p^2 + m^2 - i\epsilon}$$

# Approach 1: nonlocal field theories

Nonlocal scalar field models:

$$\mathcal{L} = \frac{1}{2}\phi e^{-\gamma\left(-\Box/M_S^2\right)}(\Box - m^2)\phi - V(\phi),$$

Ghost-free propagator:

$$\Pi(p^2) = \frac{e^{\gamma\left(p^2/M_S^2\right)}}{p^2 + m^2 - i\epsilon}$$

Perturbative unitarity (optical theorem and Cutkosky rules) holds

[Pius & Sen 2015; Briscese & Modesto 2018; Chin & Tomboulis 2018; Koshelev & Tokareva 2021; Buoninfante 2022]

# Approach 1: nonlocal QFTs of gravity

Generalized quadratic gravity action:

$$S = S_{EH} + \int d^4x \sqrt{-g}\left(R F_1(-\Box) R + R_{\mu\nu} F_2(-\Box) R^{\mu\nu} + R_{\mu\nu\rho\sigma} F_3(-\Box) R^{\mu\nu\rho\sigma} + \cdots\right)$$

Analytic form factors:

$$F_i(-\Box/M_s^2) = \sum_{n=0}^{N\leq\infty} f_{i,n}\left(\frac{-\Box}{M_s^2}\right)^n ,$$

$N = \infty \iff$ nonlocal

$M_s$: energy scale

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[Krasnikov, Kuz'min, Tomboulis, Koshelev, Siegel, Biswas, Mazumdar, Modesto, Calcagni, Briscese, Rachwal, Frolov, Zelnikov, Starobinsky, Kumar, Tokareva, Boos, Kolar, Lambiase, Buoninfante,……]

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[Asymptotic Safety Community: Saueressig, Knorr, Ripken, Platania, Schiffer, Reichert, Pawlowski, Litim, Bonanno,...]

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**See B. Knorr's talk for more general aspects of form factors in QG**

# Approach 1: nonlocal QFTs of gravity

Consider

$$S = S_{EH} + \frac{1}{2}\int d^4x\,\sqrt{-g}\big(RF_0(-\Box)R + C_{\mu\nu\rho\sigma}F_2(-\Box)C^{\mu\nu\rho\sigma}\big)$$

Propagator:

$$\Pi_{\mu\nu\rho\sigma}(k^2) = \frac{\mathcal{P}^{(2)}_{\mu\nu\rho\sigma}}{f_2(k^2)k^2} - \frac{1}{2}\frac{\mathcal{P}^{(0)}_{\mu\nu\rho\sigma}}{f_0(k^2)k^2},$$

$$f_0(k^2) = 1 + 6F_0(k^2)k^2/M_p^2$$

$$f_2(k^2) = 1 - 2F_2(k^2)k^2/M_p^2$$

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No-ghost condition:

$$f_2(k^2) = e^{-\gamma_2(k^2)}, \qquad f_0(k^2) = e^{-\gamma_0(k^2)}(1 + k^2/m_0^2)$$

Entire functions

# Approach 1: nonlocal QFTs of gravity

Ghost-free nonlocal gravity:

$$S = S_{EH} + \frac{1}{2}\int d^4x\sqrt{-g}\big(RF_0(-\Box)R + C_{\mu\nu\rho\sigma}F_2(-\Box)C^{\mu\nu\rho\sigma}\big)$$

$$F_0(-\Box) = M_p^2\,\frac{e^{-\gamma_0(-\Box)}(1-\Box/m_0^2)-1}{6\Box}, \qquad F_2(-\Box) = M_p^2\,\frac{1-e^{-\gamma_2(-\Box)}}{2\Box}$$

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Simplest case: no spin-0 dof

$$F_2(-\Box) = -3F_0(-\Box) = M_p^2\,\frac{1-e^{-\gamma_2(-\Box)}}{2\Box}$$

$$\Longrightarrow \quad \Pi_{\mu\nu\rho\sigma}(k^2) = e^{\gamma_2(k^2)}\,\Pi^{EH}_{\mu\nu\rho\sigma}(k^2)$$

# Approach 1: nonlocal QFTs of gravity

## Some remarks

- Infinite class of viable entire functions $\gamma_1(-\Box)$ and $\gamma_2(-\Box)$

- Smaller (but still infinite) class of super-renormalizable models

  [Kuz'min, Tomboulis, Modesto, Rachwal, Calcagni, Briscese, Giacchini, de Paula Netto,… ]

- Applications to black holes and compact objects

  [Biswas, Mazumdar, Siegel, Moffat, Modesto, Frolov, Zelnikov, Boos, Giacchini, de Paula Netto, Kolar, Koshelev, Lambiase, Buoninfante,….]

- Applications to the early universe cosmology (constraint $M_s \gtrsim 10^{14}$GeV)

  [Koshelev, Starobinsky, Kumar, Calcagni, Modesto, Rachwal, Tokareva,….]

# Approach 1: nonlocal QFTs of gravity

## Open issues

- Hamiltonian for nonlocal theories? (non-perturbative classical stability?)

- Dressed propagator? (non-perturbative quantum stability?)
  [Shapiro (2015)]

- Quantification of causality violation?
  [Some attempts: Tomboulis (2015); Carone (2016); Modesto (2018);
  Buoninfante, Lambiase, Mazumdar (2018)]

- Huge freedom in the choice of the entire functions !?!
  (predictivity?)

- Nonlocal Lagrangians from first principles…?

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- Nonlocal Lagrangians from first principles…?

Most important!

# Outline

- ~~**Approach 1:** correspondence principle applies (give up "locality")~~
- **Approach 2:** correspondence principle does not apply
- Discussion

# Approach 2: keep locality

Locality + (strict) renormalizability are very restrictive:

$$S = \frac{1}{2}\int d^4x\,\sqrt{-g}\left(M_p^2(R-2\Lambda)+\frac{\alpha}{6}R^2-\frac{\beta}{2}C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}\right)$$

# Approach 2: possible solutions

$$\Pi(p^2) = \frac{1}{p^2 - i\epsilon} - \frac{1}{p^2 + m_2^2 - i\varepsilon}$$

$$S^+S = 1, \qquad S = 1 + iT,$$

$$1 = \sum_{\{n\}} c_n \, |n\rangle\langle n|,$$

Optical theorem:

$$2Im\{\langle a|T|a\rangle\} = \sum_{\{n\}} c_n |\langle n|T|a\rangle|^2$$

Tree level example:

$$Im\{\langle a|T|a\rangle\} \sim \theta(p^0)[\delta(p^2) - sign(\varepsilon)\delta(p^2 + M^2)]$$

# Approach 2: possible solutions

Causal propagation & positive norms

$$\Pi(p^2) = \frac{1}{p^2 - i\epsilon} - \frac{1}{p^2 + m_2^2 - i\varepsilon}$$

$$(\epsilon > 0, \ \varepsilon > 0)$$

$$S^{+}S = 1, \qquad S = 1 + iT,$$

$$1 = \sum_{\{n\}} c_n \, |n\rangle\langle n|, \qquad c_n^{normal} > 0$$

$$c_n^{ghost} > 0$$

Optical theorem:

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Tree level example:

$$Im\{\langle a|T|a\rangle\} \sim \theta(p^0)[\delta(p^2) - sign(\varepsilon)\delta(p^2 + M^2)]$$

Unitarity is violated!

# Approach 2: possible solutions

Causal propagation & negative norms [Holdom, Salvio, Strumia,...]

$$\Pi(p^2) = \frac{1}{p^2 - i\epsilon} - \frac{1}{p^2 + m_2^2 - i\varepsilon}$$

$$(\epsilon > 0, \; \varepsilon > 0)$$

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Tree level example:

$$Im\{\langle a|T|a\rangle\} \sim \theta(p^0)[\delta(p^2) - sign(\varepsilon)\delta(p^2 + M^2)]$$

Unitarity is preserved!

# Approach 2: possible solutions

Acausal propagation & positive norms [Donoghue, Menezes,…]

$$\Pi(p^2) = \frac{1}{p^2 - i\epsilon} - \frac{1}{p^2 + m_2^2 - i\varepsilon}$$

$$(\epsilon > 0,\ \varepsilon < 0)$$

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$$Im\{\langle a|T|a\rangle\} \sim \theta(p^0)[\delta(p^2) - sign(\varepsilon)\delta(p^2 + M^2)] \geq 0$$

Unitarity is preserved!

# Approach 2: Some Remarks

## Beyond tree-level

The massive spin-2 ghost gets a width: [Donoghue & Menezes 2018+]

$$\frac{-1}{p^2 + m_2^2 + i\varepsilon} \quad \longrightarrow \quad \frac{-1}{p^2 + m_{2,ph}^2 + i(\varepsilon + m_{2,ph}\Gamma)}$$

$(\varepsilon \geq 0$ anti-Feynman$)$ $\qquad (\Gamma \sim m_2^3/M_p^2 \geq 0)$

Ghost life-time: $\tau_{decay} \sim 1/\Gamma \sim M_p^2/m_2^3$

If $m_2 > 2m_0$ & $m_0 \sim 10^{13} GeV \quad \Longrightarrow \quad \tau_{decay} \lesssim 10^{-3} GeV^{-1} \sim 10^{-28}\text{sec}$

# Approach 2: Some Remarks

Beyond tree-level

The massive spin-2 ghost gets a width: [Donoghue & Menezes 2018+]

$$\frac{-1}{p^2+m_2^2+i\varepsilon} \quad \longrightarrow \quad \frac{-1}{p^2+m_{2,ph}^2+i(\varepsilon+m_{2,ph}\Gamma)}$$

($\varepsilon \geq 0$ anti-Feynman) ($\Gamma \sim m_2^3/M_p^2 \geq 0$)

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Ghost must be projected away from the physical Hilbert space

[Veltman projection (1964)]

# Approach 2: Some Remarks

Classical limit of Quadratic Gravity ?

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- Is just a low-energy limit $(E \ll m_2)$ ? I don't think so.

# Approach 2: Some Remarks

Classical limit of Quadratic Gravity ?

Two time-scale regimes to consider (in my opinion):

1. $\Delta t \geq \tau_{decay}$:

Classical limit must be taken consistently with the ‘‘quantum projection’’

2. $\Delta t < \tau_{decay}$:

Ghost is still alive and can propagate, no projection

[may it be related to the high-energy limit $(E \gg m_2)$?]

# Approach 2: Some Remarks

Tree-level 2-2 graviton scattering amplitude

Despite renormalizability, it's the same as Einstein's general relativity:

[Modesto et al. (2015); Holdom (2021)]

$$\mathcal{M}_{2-2}(s) \sim G\, s \sim E^2/M_p^2$$

Perturbativity (not unitarity) breaks at $E \sim M_p$

This may indicate that non-perturbative effects must be taken into account (e.g. black-hole formation !)

# Approach 2: Some Remarks

Implications of a non-zero (positive) cosmological constant?

$$S = \frac{1}{2}\int d^4x\,\sqrt{-g}\left(M_p^2(R-2\Lambda) + \frac{\alpha}{6}R^2 - \frac{\beta}{2}C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}\right)$$

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Simple question (non-trivial answer):

[Buoninfante 2308.11324]

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$$\beta \longrightarrow \infty \qquad \Longrightarrow \qquad \begin{aligned} &\Lambda = 0: \quad m_2^2 = \frac{M_p^2}{\beta} \longrightarrow 0 \\ &\Lambda \neq 0: \quad m_2^2 = \frac{M_p^2}{\beta} + \frac{2}{3}\Lambda\left(2\frac{\alpha}{\beta}+1\right) \longrightarrow \frac{2}{3}\Lambda \end{aligned}$$

# Approach 2: Some Remarks

$$\Lambda > 0: \qquad \lim_{\beta \to \infty} S = \; ?$$

- Massless spin-2 & $\pm 2, \pm 1$ helicities of massive spin-2 ghost decouple
- Massive spin-0 ($\phi$) & helicity-0 ($\chi$) of spin-2 ghost survive

[Buoninfante 2308.11324]

$$S_{\phi\chi}[g, \phi, \chi] = \int d^4x \sqrt{-g} \left[ \frac{1}{2} \left( \partial_\mu \chi \partial^\mu \chi - \partial_\mu \phi \partial^\mu \phi \right) - V(\phi, \chi) \right]$$

$$V(\phi, \chi) = \frac{\Lambda}{36 \bar{M}_p^2} \left( \chi^2 - \phi^2 - 6 \bar{M}_p^2 \right)^2 + \frac{m_0^2}{12 \bar{M}_p^2} \phi^2 (\chi + \phi)^2$$

$$\bar{M}_p^2 = M_p^2 + \frac{4}{3} \alpha \Lambda$$

# Summary

## Ghost puzzle in Perturbative Quantum gravity

**Approach 1:** give up locality and kill the ghost at the classical level

- Correspondence principle still holds
- Bigger price to pay: uniqueness is lost (predictivity?)

**Approach 2:** keep locality + strict renormalizability

- Unique Lagrangian
- Correspondence principle does <u>not</u> hold
- Some open questions: classical limit; non-perturbative stability; more investigations in curved spacetimes

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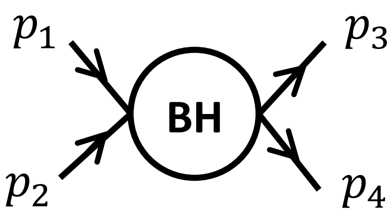
# Where to expect nonlocality?

Black hole formation?

Usual argument: non-perturbative 2-2 scattering process [Giddings, Dvali,……]

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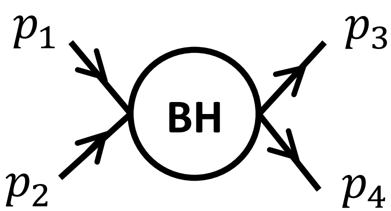


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Violates the <u>Cerulus-Martin bound</u> [assumes locality (polynomial boundedness)]

[Cerulus & Martin (1964)]

$$|\mathcal{M}_{2-2}(s)| \gtrsim e^{-c\sqrt{s}\log s}$$

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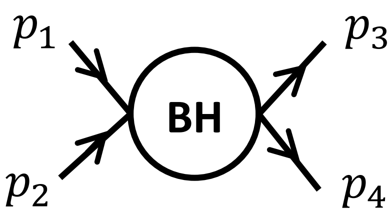


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More general bound [includes nonlocality (exponential boundedness)]:

[Buoninfante, Tokuda, Yamaguchi (2023)]

$$\begin{cases} \alpha = 0: & \text{local} \\ 0 < \alpha < 1/2: & \text{strictly localizable} \\ \alpha \geq 1/2: & \text{nonlocalizable} \end{cases}$$

$$|\mathcal{M}_{2-2}(s)| \gtrsim e^{-c_1\sqrt{s}\log s - c_2 s^{\alpha+\frac{1}{2}}}$$

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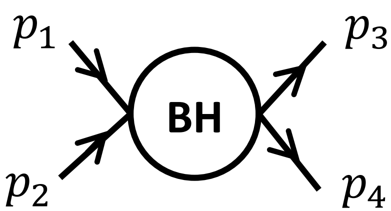


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IF black holes form as above $\Longrightarrow$ "nonlocal bound" on gravity:

$$\alpha \geq \frac{1}{2}$$

Thank you for your attention!