Quantum Hair
and
Information Loss Paradox

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Based on a series of works in collaboration with S. Hsu
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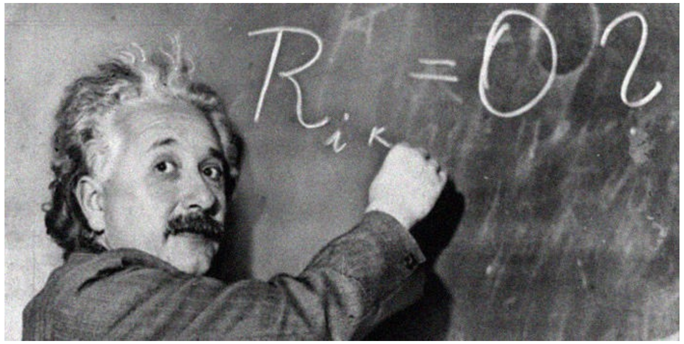
Take-home message

- We have identified quantum gravitational corrections to the external metric of compact objects which contain information about the interior metric. This is a new quantum hair.
- We have shown how to apply these results to black holes.
- This quantum hair leads to a solution to the information paradox.
- Using EFT techniques, we are able to calculate quantum gravitational corrections to the Hawking amplitudes.
- We can also show that the black hole spectrum differs from that of a black body and it thus contains information.



Unique effective action for quantum gravity

The Hilbert-Einstein action

$$S = \int d^4x \sqrt{-g} \frac{2}{\kappa^2} R$$

receives corrections when integrating out fluctuations of the graviton (and any other matter fields depending on the energy under consideration), one obtains:

$$S = \int d^4x \sqrt{-g} \left[\left(\frac{1}{2} M^2 + \xi H^\dagger H \right) R - \Lambda_C^4 + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + c_4 \Box R \right.$$
$$\left. - b_1 R \log \frac{\Box}{\mu_1^2} R - b_2 R_{\mu\nu} \log \frac{\Box}{\mu_2^2} R^{\mu\nu} - b_3 R_{\mu\nu\rho\sigma} \log \frac{\Box}{\mu_3^2} R^{\mu\nu\rho\sigma} + \mathcal{O}(M_\star^{-2}) + \mathcal{L}_{SM} \right]$$

The non-local part of the EFT

- The Wilson coefficients of the non-local operators are universal predictions of quantum gravity:

$$-b_1 R \log \frac{\Box}{\mu_1^2} R - b_2 R_{\mu\nu} \log \frac{\Box}{\mu_2^2} R^{\mu\nu} - b_3 R_{\mu\nu\rho\sigma} \log \frac{\Box}{\mu_3^2} R^{\mu\nu\rho\sigma}$$

	b_1	b_2	b_3
Scalar	$5(6\xi-1)^2$	-2	2
Fermion	-5	8	7
Vector	-50	176	-26
Graviton	250	-244	424

NB: they are calculated using dim-reg.

All numbers should be divided by 11520 π^2

- The Wilson coefficients of the local operators on the other hand are not calculable: this is the price to pay.

Quantum Corrections to static star and quantum hair

- We start from the unique effective action.

- Field equations
$$\mathcal{R}_{\mu\nu} - \frac{1}{2}\mathcal{R}\, g_{\mu\nu} + 16\,\pi\, G_{\rm N}\left(H^{\rm L}_{\mu\nu} + H^{\rm NL}_{\mu\nu}\right) = 8\,\pi\, G_N T_{\mu\nu}\ .$$

- Consider the corrections to the metric of a stationary homogeneous and isotropic star with radius R_S and density
$$\rho(r) = \rho_0\,\Theta(R_{\rm s} - r) = \begin{cases} \rho_0 & \text{if } r < R_{\rm s} \\ 0 & \text{if } r > R_{\rm s}\ , \end{cases}$$

- The solution to the Einstein equation inside this star (for $r \leq R_S$) is the well-known interior Schwarzschild metric

$$\mathrm{d}s^2 = \left(3\sqrt{1-\frac{2\,G_{\rm N}\,M}{R_{\rm s}}} - \sqrt{1-\frac{2\,G_{\rm N}\,M\,r^2}{R_{\rm s}^3}}\right)^2\frac{\mathrm{d}t^2}{4} - \left(1-\frac{2\,G_{\rm N}\,Mr^2}{R_{\rm s}^3}\right)^{-1}\mathrm{d}r^2 - r^2\,\mathrm{d}\Omega^2 \ \equiv g^{\rm int}_{\mu\nu}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu\ ,$$

- where the Misner Sharp mass.

- Outside the star

$$ds^2 = \left(1 - \frac{2\,G_{\rm N}\,M}{r}\right) dt^2 - \left(1 - \frac{2\,G_{\rm N}\,M}{r}\right)^{-1} dr^2 - r^2\,d\Omega^2 \equiv g^{\rm ext}_{\mu\nu}\,dx^\mu\,dx^\nu$$

- Use perturbation theory around classical metric $\tilde{g}_{\mu\nu} = g_{\mu\nu} + g^{\rm q}_{\mu\nu}$

- Quantum correction outside the star

$$\begin{aligned} g_{tt} &= 1 - \frac{2G_N M}{r} \\ &\quad -128\pi^2(\alpha+\beta+3\gamma)\frac{l_p^2}{r^2}\left[\frac{G_N M}{r}\left(1 + \frac{3R_s^2}{5r^2} + \mathcal{O}(R_s/r)^4\right) + \mathcal{O}(G_N M/r)^2\right] \\ &\quad + \mathcal{O}(l_p/r)^4 \ , \end{aligned}$$

where l_P is the Planck length.

- The coefficient of the r^{-5} term is proportional to $G_N^2 M\, R_S^{-3}$: i.e., it is a quantum gravitational effect proportional to the density of source object.

- Our result can be extended to a time dependent collapse (Phys.Rev.D 108 (2023) 8, 086012): replace R_s by the time-dependence via the radius of the star $R_s(t)$.

- For a distant observer, $r \gg R_s(t)$ at all times, we can expand the correction to the metric, and the r^{-5} dependence remains during the totality of the collapse.

- Eventually, $R_s(t)$ will reach $2G_N M$ and a closed trapped surface will form indicating the formation of a black hole.

- An observer could in principle measure the coefficient of the r^{-5} correction to the metric.

- This correction contains information about the matter distribution that collapsed and could thus enable the observer to differentiate between black holes formed by different matter distributions.

- Quantum gravity produces a new kind of hair on black holes and we have shown that this hair affects Hawking's evaporation amplitudes.

Asymptotic Quantum States of the graviton field

- Consider a compact source which is an energy eigenstate with eigenvalue E.

- Graviton quantum state $\psi_g(E)$ is exactly analogous to the quantum state of the $U(1)$ vector field (Coulomb potential) created by a charge Q. This is a coherent state

$$|0\rangle_Q = \exp\left[Q \int d^3k\, q(k) b^\dagger(k)\right] |0\rangle_{Q=0}$$

- where $b^\dagger(k)$ is a linear combination of creation operators of the non-propagating (temporal + longitudinal, depending on gauge) modes of the photon.

- For gravity Q is replaced by the energy eigenvalue of the source state and the coherent state modes are temporal and longitudinal graviton modes.

- In both gauge theory and gravity the asymptotic state is determined by Gauss law via constrained quantization.

- $\psi_g(E)$ depends on E: each distinct energy eigenstate of the compact source has a different graviton quantum state.

- A semiclassical matter configuration is a superposition of energy eigenstates with support concentrated in some narrow band of energies.

$$\psi = \sum_n c_n \psi_n$$

where ψ_n are energy eigenstates with eigenvalues E_n.

- Resulting gravity state is a superposition state: $$\psi_g = \sum_n c_n \psi_g(E_n)$$

- In QED: long wavelength photons couple to total charge of composite object.

- In gravity: long wavelength gravitons couple to energy eigenvalue of compact object.

- Semiclassical compact sources produce external graviton states which are complex superpositions given by ψ_g.

- The stars considered before are a concrete example of this general result.

A Brief History of Hawking's Information Paradox

- The thermal nature of his radiation led Stephen Hawking in 1976 to argue that black holes would destroy quantum information.
- In other words, black holes cause pure states to evolve into mixed states.
- Mathur’s formulation is particularly clear: his analysis tracks the entanglement entropy of Hawking radiation emitted on a nice slice.
- Nice slices are spacelike surfaces which intersect both the interior of the black hole and the emitted Hawking radiation.
- We shall briefly review Mathur's formulation of the paradox.

- Let us call the modes outside the horizon b_1, b_2, ..., and those inside the horizon e_1, e_2, ... The initial slice in the nice slice foliation contains only the matter state $|\psi\rangle_M$ (i.e., the black hole), and none of the e_i, b_i.

- The first step of evolution stretches the spacelike slice, so that the particle modes e_1, b_1 are now present on the new slice. The state of these modes has the schematic form

$$\frac{1}{\sqrt{2}}\left(|0\rangle_{e_1}|0\rangle_{b_1} + |1\rangle_{e_1}|1\rangle_{b_1}\right)$$

  where the numbers 0, 1 give the occupation number of a particle mode.

- The entanglement of the state outside the horizon (given by the mode b_1) with the state inside (given by the mode e_1) yields $S_{\text{entanglement}} = \ln 2$.

- In the next step of evolution the modes b_1, e_1 at the earlier step move apart, and in the region between them there appears another pair of modes b_2, e_2 in a state that has the same form as the first state.

- After N steps we have the state

$$
\begin{aligned}
|\Psi\rangle \approx \ |\psi\rangle_M \ & \otimes \ \left(\frac{1}{\sqrt{2}}|0\rangle_{e_1}|0\rangle_{b_1} + \frac{1}{\sqrt{2}}|1\rangle_{e_1}|1\rangle_{b_1}\right) \\
& \otimes \ \left(\frac{1}{\sqrt{2}}|0\rangle_{e_2}|0\rangle_{b_2} + \frac{1}{\sqrt{2}}|1\rangle_{e_2}|1\rangle_{b_2}\right) \\
& \ldots \\
& \otimes \ \left(\frac{1}{\sqrt{2}}|0\rangle_{e_N}|0\rangle_{b_N} + \frac{1}{\sqrt{2}}|1\rangle_{e_N}|1\rangle_{b_N}\right) \ .
\end{aligned}
$$

- The initial matter state appears in a tensor product with all the other quanta, since the pair creation happens far away. There is no connection to the matter state in the leading order Hawking process.

- The modes b_i are entangled with the e_i and the black hole with an entropy

$$S_{\text{entanglement}} = N \ln 2.$$

- This entanglement grows by ln 2 with each succeeding emission.

- If the black hole evaporates away completely, the b_i quanta outside will be in an entangled state, but there will be nothing that they are entangled with.

- The initial pure state $|\psi\rangle_M$ has evolved to a mixed state, described by a density matrix.

- Mathur argues further that small corrections ε to Hawking evaporation cannot change this qualitative result: the entanglement entropy increases by $\approx \ln 2 - \varepsilon$ with each emitted quantum.

- This formulation is limited by the assumption of a semiclassical spacetime background.

- Specifically, as we elaborate in what follows, it does not address the possibility of entanglement between different background geometries (gravitational states).

- We will argue that black holes (their geometry) and the Hawking radiation form a macroscopic superposition

Quantum Hair and Unitary Evaporation.

- We shall now describe our solution to Hawking's paradox.
- As we have seen the analysis of the state of the graviton field produced by a compact matter source (e.g., a black hole) revealed the following:

1. The asymptotic graviton state of an energy eigenstate source is determined at leading order by the energy eigenvalue and it can be expressed explicitly as a coherent state which depends on this eigenvalue.

Insofar as there are no accidental energy degeneracies there is a one-to-one map between graviton states and matter source states.

A semiclassical matter source produces an entangled graviton state.

2. Quantum gravitational fluctuations (i.e., graviton loops) produce corrections to the long range potential (e.g., $\sim r^{-5}$) whose coefficients depend on the internal state of the source.

This provides an explicit example of how the graviton quantum state (corresponding to the semiclassical potential) encodes information about the internal state of a black hole. Note the calculation is insensitive to short distance effects in quantum gravity.

Black hole radiation

- Using these properties above, we can write the quantum state of the exterior metric (equivalently, the quantum state of the exterior geometry) as

$$\Psi_i = \sum_n c_n \Psi_g(E_n) = \sum_n c_n \,|\, g(E_n) \,\rangle \ .$$

- A semiclassical state has support concentrated in some range of energies E, where the magnitudes of c_n are largest. For simplicity, when representing the exterior metric state g(E) we only write the energy explicitly and suppress the other quantum numbers.

- Assume for convenience that the black hole emits one quantum at a time (e.g., at fixed intervals), culminating in a final state of N radiation quanta:

$$| r_1 \, r_2 \, r_3 \, \cdots \, r_N \rangle \, .$$

- The quantum numbers of the i-th emitted radiation particle include the energy Δ_i, momentum p_i, spin s_i, charge q_i, etc.

- The symbol r_i is used to represent all of these values:

$$r_i \sim \{\Delta_i, p_i, s_i, q_i, \ldots\} \, .$$

- A final radiation state is specified by the values of $\{r_1, r_2, \ldots, r_N\}$.

- $\alpha(E, r_i)$: amplitude for emission of quantum r_i from exterior metric state $\Psi_g(E)$.

- At leading order, it is just the semiclassical Hawking amplitude for a black hole of mass E.

Remarks on $\alpha(E, r_i)$:

- In the leading approximation the amplitudes are those of thermal emission, but at subleading order (i.e., $\sim S^{-k}$ for perturbative corrections, or $exp(-S)$ for nonperturbative effects, where S is the black hole entropy) additional dependence on (E, r_i) will emerge.

- The fact that these corrections can depend on the internal state of the black hole is a consequence of quantum hair.

- It has been shown that even corrections as small as $exp(-S)$ can purify a maximally mixed Hawking state (i.e., can perturb the radiation density matrix ρ so that tr $\rho^2 = 1$), because the dimensionality ($\sim exp\ S$) of the Hilbert space is so large. (arXiv1211.6767 (Padadodimas and Raju) and arXiv:1308.5686 (Hsu))

Unitarity black hole evolution

- When the black hole emits the first radiation quantum r_1 it evolves into the exterior state given on the right below:

$$\Psi_i = \sum_n c_n \,|\, g(E_n)\,\rangle \;\rightarrow\; \sum_n \sum_{r_1} c_n\, \alpha(E_n, r_1)\,|\, g(E_n - \Delta_1), r_1\rangle \;.$$

- In this notation g refers to the exterior geometry and r_1 to the radiation. The next emission leads to

$$\sum_n \sum_{r_1, r_2} c_n\, \alpha(E_n, r_1)\, \alpha(E_n - \Delta_1, r_2)\,|\, g(E_n - \Delta_1 - \Delta_2), r_1, r_2\rangle$$

- and the final radiation state is

$$\sum_n \sum_{r_1, r_2, \ldots, r_N} c_n\, \alpha(E_n, r_1)\, \alpha(E_n - \Delta_1, r_2)\, \alpha(E_n - \Delta_1 - \Delta_2, r_3) \cdots |\, r_1\, r_2 \cdots r_N\rangle\,.$$

- Final radiation state:

$$\sum_{n} \sum_{r_1, r_2, \ldots, r_N} c_n \, \alpha(E_n, r_1) \, \alpha(E_n - \Delta_1, r_2) \, \alpha(E_n - \Delta_1 - \Delta_2, r_3) \cdots |r_1 \, r_2 \cdots r_N\rangle \, .$$

- It is not a tensor product!

- In the final state we omit reference to the geometry g as the black hole no longer exists: there is no horizon and the spacetime is approximately flat.

- The final radiation state is a complex superposition which depends linearly on the initial black hole state.

- The final radiation state is a macroscopic superposition state.

- Under time reversal, the radiation state evolves back to the original black hole quantum state.

- Quantum hair provides a mechanism by which the amplitudes $\alpha(E, r)$ can depend on the internal state.

- The result is manifestly unitary, and the final state is manifestly a pure state:
  - For each distinct initial state there is a different final radiation state.
  - The time-reversed evolution of a final radiation state results in a specific initial state.

- These results provide, for the first time, a physical picture of how the information is encoded in the Hawking radiation.

Quantum Gravitational Corrections to Particle Creation by Black Holes

- We revisit Hawking's original calculation adding quantum gravitational corrections to the background metric.

- For a Schwarzschild black hole, the leading quantum gravitational correction to the metric appears at third order in curvature (see 2108.06824)

$$ds^2 = -f(r)dt^2 + \frac{1}{g(r)}dr^2 + r^2 d\Omega^2 \, ,$$

$$f(r) = 1 - \frac{2G_N M}{r} + 640\pi c_6 \frac{G_N^5 M^3}{r^7} \, ,$$

$$g(r) = 1 - \frac{2G_N M}{r} + 128\pi c_6 \frac{G_N^4 M^2}{r^6}\left(27 - 49\frac{G_N M}{r}\right) \, .$$

- The field operator is given by $\Phi = \sum_i \left(f_i \mathbf{a_i} + \bar{f}_i \mathbf{a_i}^\dagger \right) = \sum_i \left(p_i \mathbf{b_i} + \bar{p}_i \mathbf{b_i}^\dagger + q_i \mathbf{c_i} + \bar{q}_i \mathbf{c_i}^\dagger \right)$
- Amplitude in terms of the Bogoliubov coefficients reads

$$p_\omega = \int_0^\infty \left(\alpha_{\omega\omega'} f_{\omega'} + \beta_{\omega\omega'} \bar{f}_{\omega'} \right) d\omega',$$

where $$\alpha_{\omega\omega'} = -iKe^{i\omega' v_0} e^{\left(2\pi M G_N - \frac{4\pi^2 c_6}{M^3 G_N}\right)\omega} \times \int_{-\infty}^{0} dx \left(\frac{\omega'}{\omega}\right)^{1/2} e^{\omega' x} \exp\left[i\omega \left(4MG_N - \frac{8\pi c_6}{M^3 G_N}\right) \ln\left(\frac{|x|}{CD}\right)\right]$$

and

$$\beta_{\omega\omega'} = iKe^{-i\omega' v_0} e^{-\left(2\pi M G_N - \frac{4\pi^2 c_6}{M^3 G_N}\right)\omega} \times \int_{-\infty}^{0} dx \left(\frac{\omega'}{\omega}\right)^{1/2} e^{\omega' x} \exp\left[i\omega \left(4MG_N - \frac{8\pi c_6}{M^3 G_N}\right) \ln\left(\frac{|x|}{CD}\right)\right] .$$

- The quantum amplitude depends on the quantum correction to the metric: the quantum hair induces a quantum correction to the amplitude for production of Hawking radiation.
- This is the mechanism by which information escapes the black hole.

- We now want to take energy conservation into account: Parikh and Wilczek tunneling method.

- The emission spectrum we obtain deviates from that of a black body:

$$F(\omega) = \frac{d\omega}{2\pi} \frac{1}{e^{8\pi\omega M G_N \left(1-\frac{\omega}{2M}\right)\left(1-\frac{2\pi c_6}{G_N^2 M^2 (M-\omega)^2}\right)} - 1}$$

- These results show that there is information about the interior state in the radiation from a black hole.

- Not only do the Hawking amplitudes depend on the quantum corrections to the metric (quantum hair), but the power spectrum also depends on these corrections and it does not match that of a black body once energy conservation and quantum gravitational effects are taken into account.

- The alpha's in our equations for the unitary evolution of black holes are calculable

Conclusions

- We have identified a new kind of quantum hair.
- We present a generic expression for the quantum state of the graviton field.
- We then used these results to reconsider black hole radiation and evolution showing that it is a unitary process.
- We have calculated the leading order quantum gravitational corrections to Hawking amplitude and spectrum. We find a deviation from the black body spectrum.

Thanks for your attention