# Form Factors - a unifying language for Quantum Gravity

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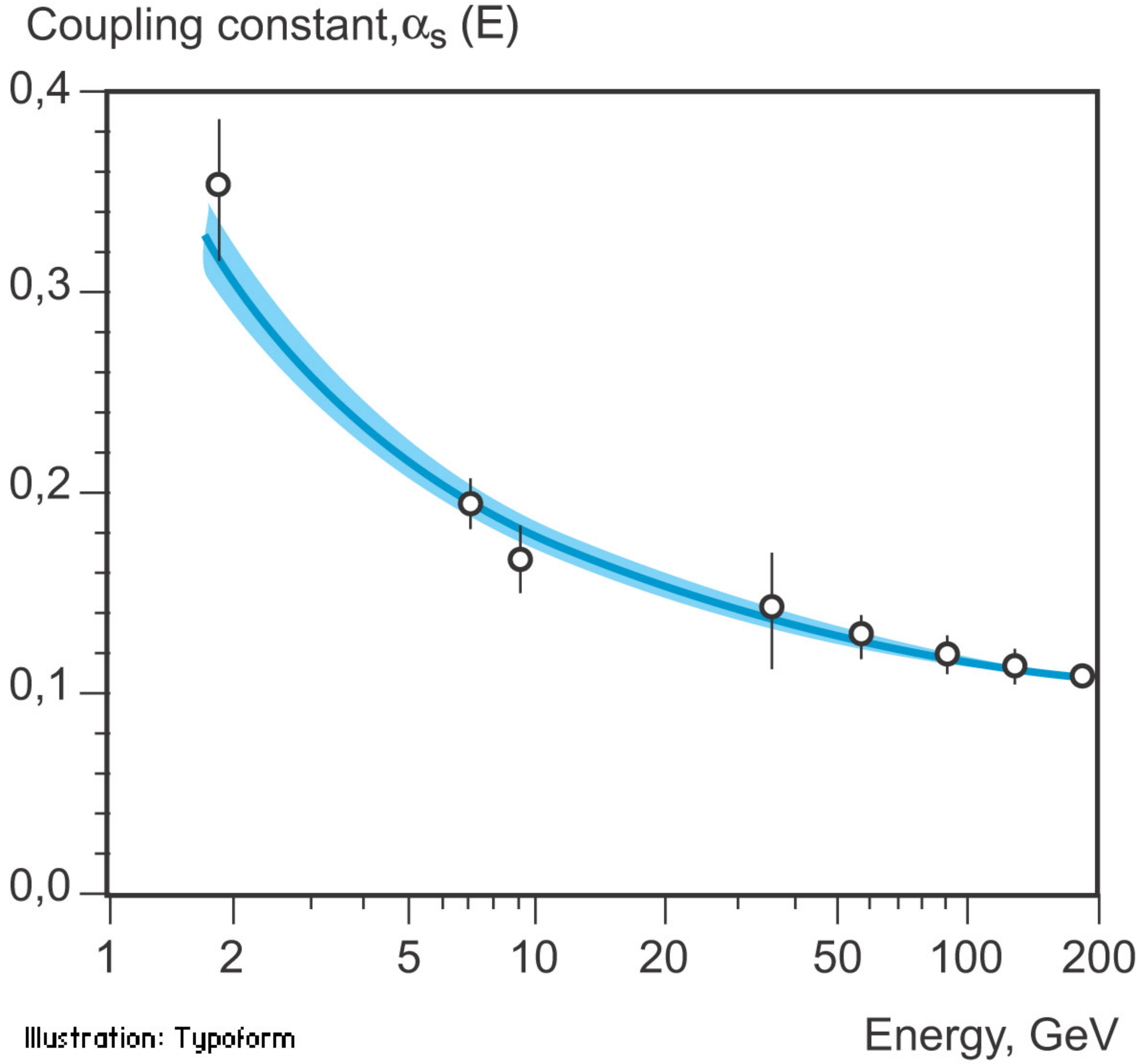
# Outline

- Running couplings in a covariant theory - form factors
- Gravity-mediated scattering amplitudes
- Form factors (and the effective action) as a universal language

# Running couplings in a covariant theory - form factors

# Running coupling constants

- established experimental fact: coupling constants "run with energy"




**Nobel prize in Physics 2004**
**(Gross, Politzer, Wilczek)**
**"for the discovery of asymptotic freedom in the theory of the strong interaction"**

nobelprize.org

# Running coupling constants

- established experimental fact: coupling constants "run with energy"
- measure scattering cross sections and compare them to theoretical predictions - coupling "constants" depend on energy scale dictated by their beta functions

$$\beta_{\alpha_s} = -\left(11 - \frac{2}{3}N_f\right)\frac{\alpha_s^2}{2\pi} + \mathcal{O}(\alpha_s^3)$$

# Running coupling constants

- What is the fundamental meaning of “running coupling constants”?
  - “fundamental”: discuss in terms of QFT concepts using the language of the effective action $\Gamma$
- How do we generalise this notion to a curved spacetime?

# Form Factors

- RG running = dependence of a coupling in the effective action on covariant derivatives

$$\text{EM/YM:} \qquad \Gamma = \int \mathrm{d}^4x \, \sqrt{-g} \left[ -\frac{1}{4} \mathcal{F}^{\mu\nu} \frac{1}{\alpha_s(\Box)} \mathcal{F}_{\mu\nu} + \mathcal{O}(\mathcal{F}^3) \right]$$

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$$\text{gravity:} \qquad \Gamma = \frac{1}{16\pi G_N}\int \mathrm{d}^4x\,\sqrt{-g}\left[2\Lambda - R + R\,f_R(\Box)\,R + C^{\mu\nu\rho\sigma}f_C(\Box)C_{\mu\nu\rho\sigma} + \mathcal{O}(\mathcal{R}^3)\right]$$

# Form Factors

- interaction terms are more complicated, e.g. three-point function:

$$\Gamma^{(3)} \supset \frac{1}{16\pi G_N} \int \mathrm{d}^4x \sqrt{-g}\, f_{R^3}(\Box_1, \Box_2, \Box_3)\, R\, R\, R$$

- four-point function and higher: operator ordering needs convention (difference is of higher order)

$$\Gamma^{(4)} \supset \frac{1}{16\pi G_N} \int \mathrm{d}^4x \sqrt{-g}\, f_{R^4}\left(\{-D_i \cdot D_j\}\right)\, R\, R\, R\, R$$

# Form Factors

- RG running of couplings generically depends on several momentum scales - there is no unique scale in many processes

**see also discussion in 2307.00055 (Buccio, Donoghue, Percacci)**

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- based on curvature/field strength expansion - can access momentum dependence by considering n-point function around vanishing field configuration

**BK-Ripken-Saueressig collaboration: 1907.02903, 2111.12365, 2210.16072**

# Gravity-mediated scattering amplitudes

# Graviton-mediated scattering amplitudes

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- first non-trivial example: compute $2 \to 2$ gravitational scattering amplitudes and confront them with theoretical&experimental constraints
- benefits:
  - probe quantum gravity effects
  - direct link to observables
  - independent of arbitrary choices
  - use effective action = tree-level diagrams encode “everything”

# Graviton-mediated scattering amplitudes

- strategy for a given scattering amplitude:

  1. parameterise all possible terms in the effective action that contribute to the scattering event

  2. compute ingredients from first principles

  3. confront with experimental data and theoretical constraints (finiteness, unitarity, causality, …)

**recall Andrew Tolley's talk**

# Graviton-mediated scattering amplitudes

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**Form Factors!**

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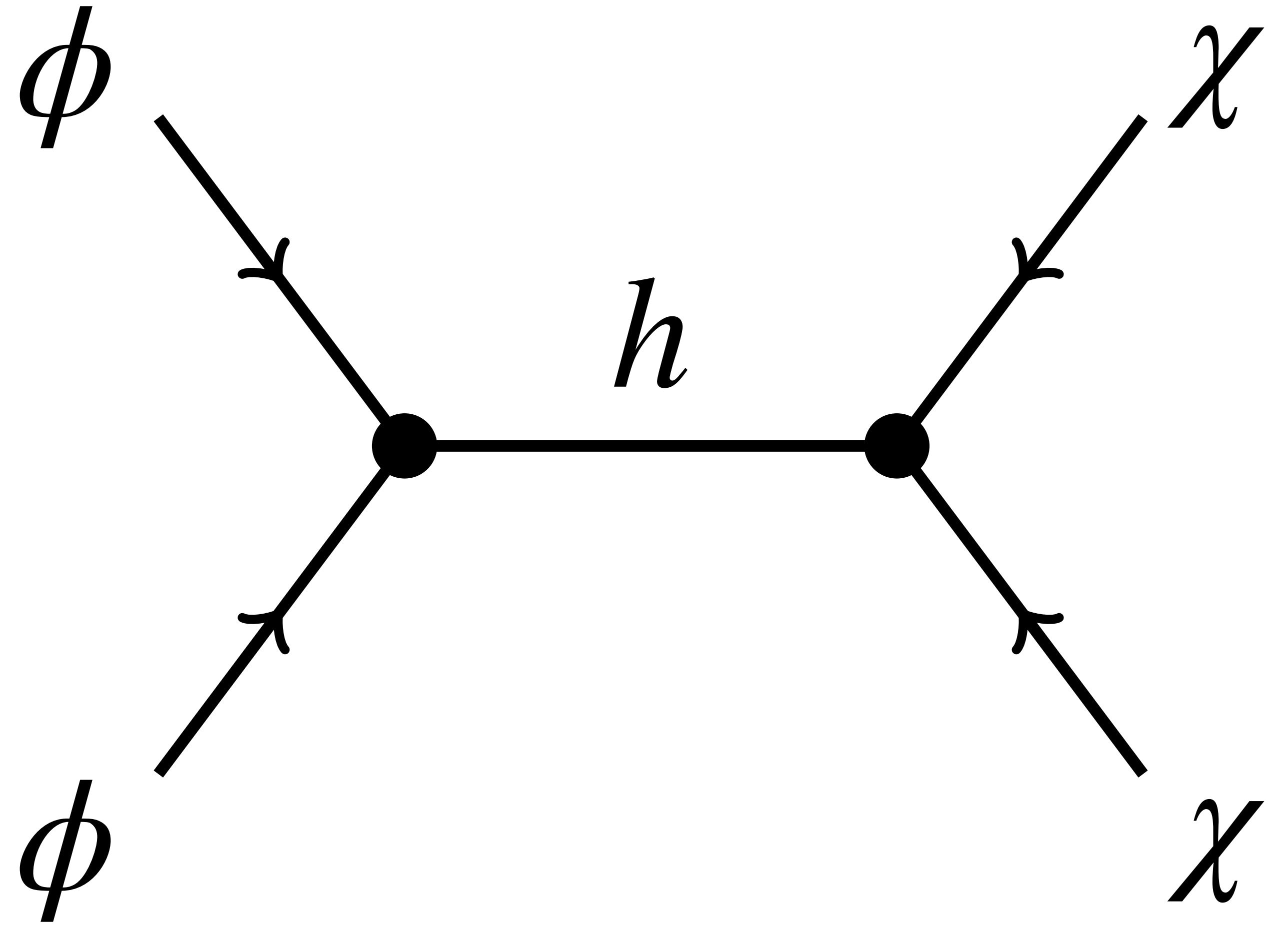
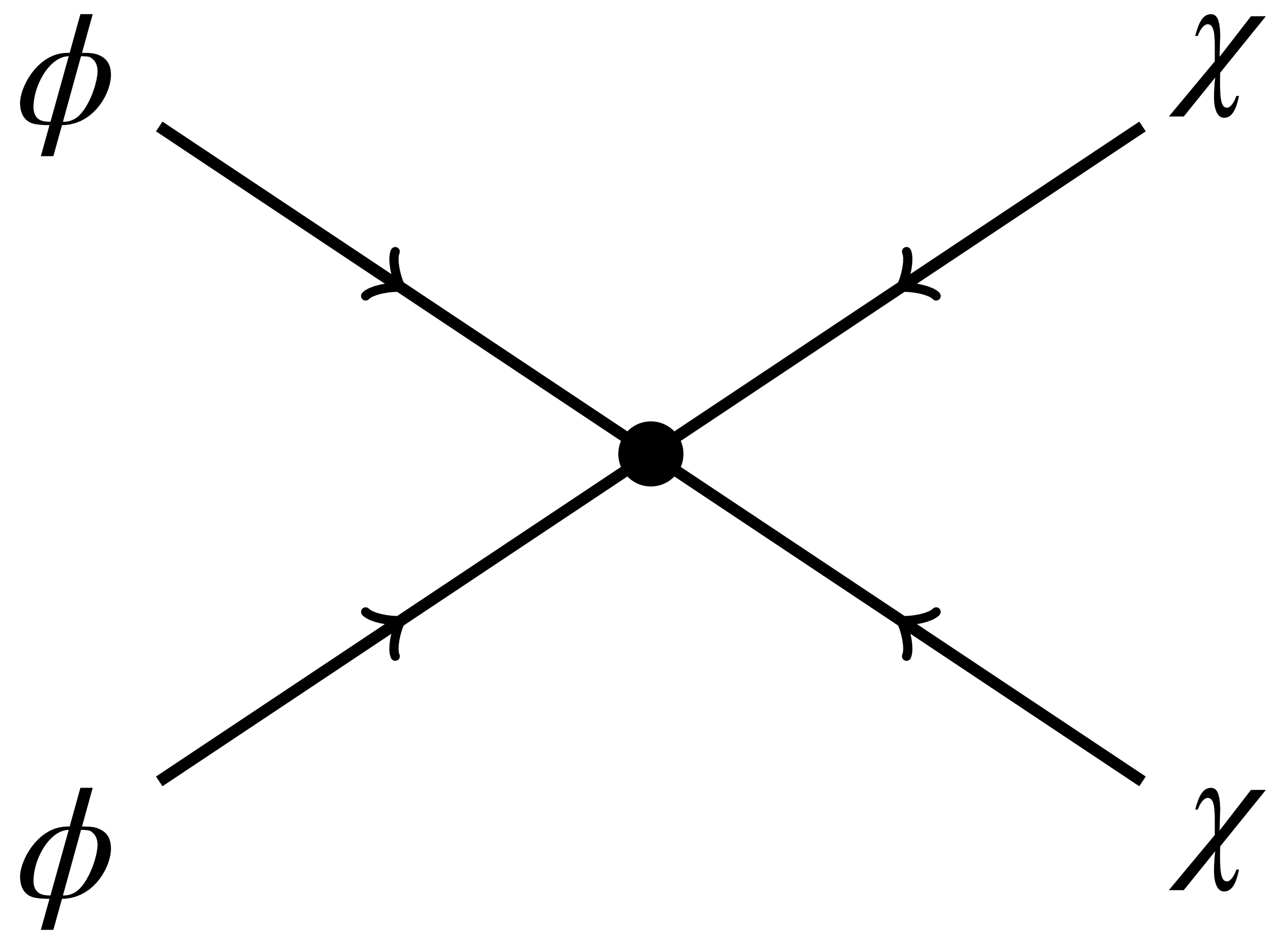
## *Parameterisation of amplitudes with form factors*

# Scalar-scalar scattering

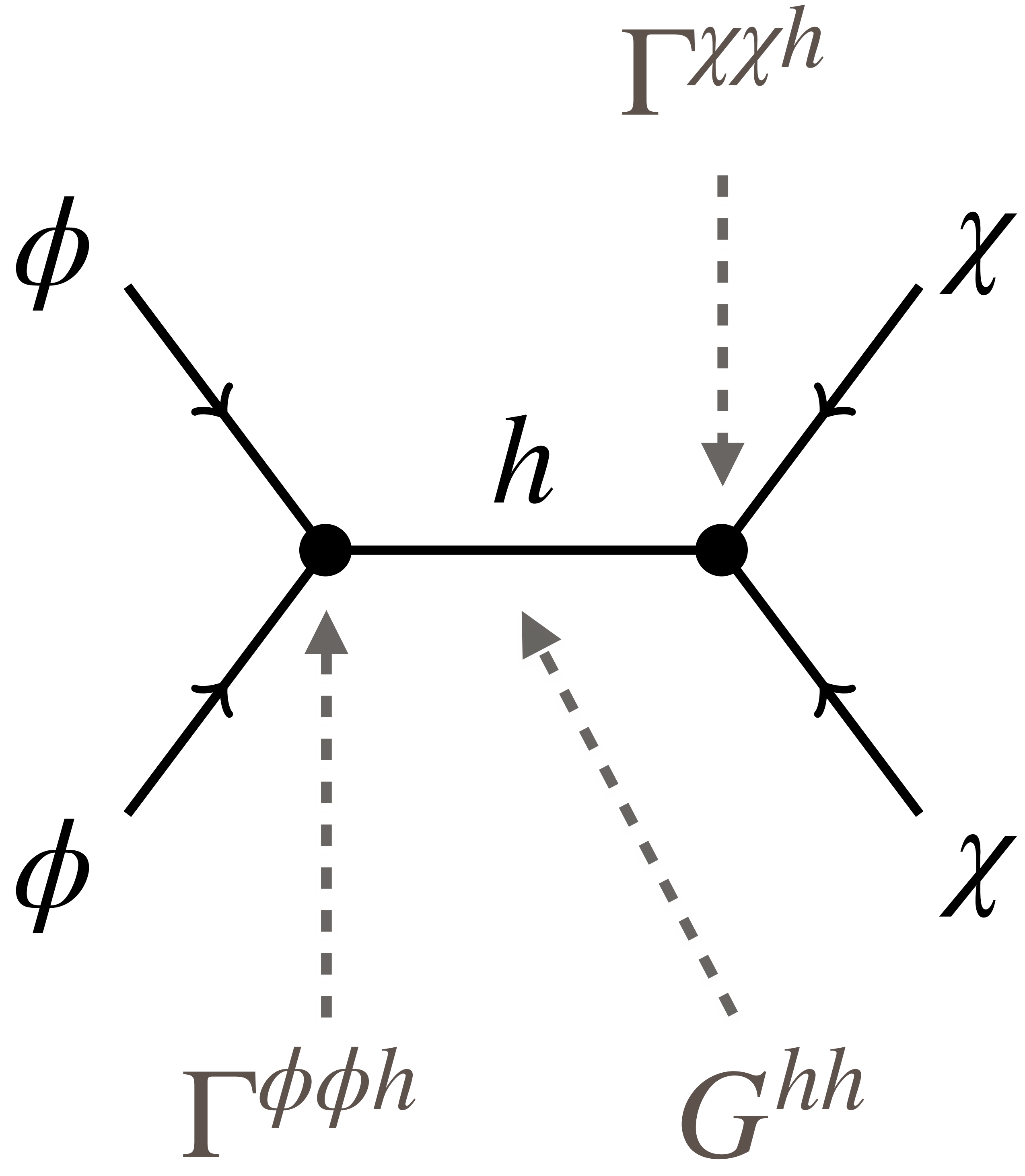
- gravity-mediated scalar scattering:

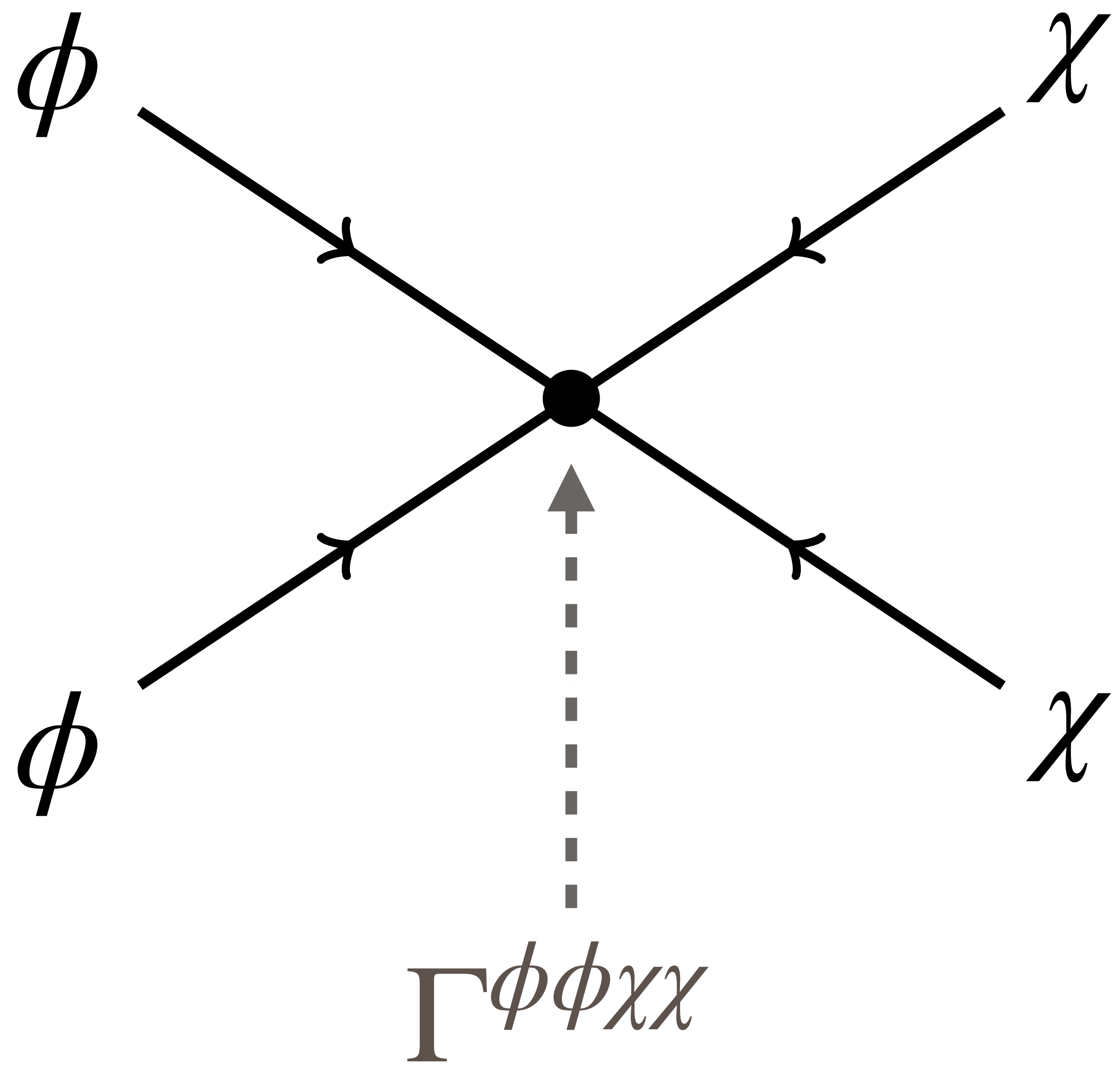


# Scalar-scalar scattering

- gravity-mediated scalar scattering:



# Scalar-scalar scattering

- necessary ingredients in the effective action:

$G^{hh}$

$$\Gamma \simeq \frac{1}{16\pi G_N}\int \mathrm{d}^4x\,\sqrt{-g}\,[-R + R\,f_R(\Box)\,R + C^{\mu\nu\rho\sigma}\,f_C(\Box)C_{\mu\nu\rho\sigma}]$$
$$+\int \mathrm{d}^4x\,\sqrt{-g}\left[\frac{1}{2}\phi\,f_\phi(\Box)\,\phi + f_{R\phi\phi}(\Box_1,\Box_2,\Box_3)R\,\phi\,\phi + f_{Ric\phi\phi}(\Box_1,\Box_2,\Box_3)R^{\mu\nu}\,(D_\mu D_\nu\phi)\,\phi\right] + (\phi\to\chi)$$
$$+\frac{1}{(2!)^2}\int \mathrm{d}^4x\,\sqrt{-g}\,f_{\phi\chi}(\{-D_i\cdot D_j\})\,\phi\,\phi\,\chi\,\chi$$

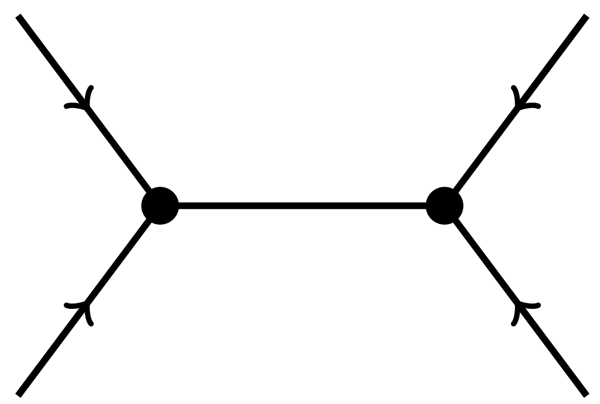
$\Gamma^{\phi\phi h}$ $\Gamma^{\chi\chi h}$

$\Gamma^{\phi\phi\chi\chi}$

***full momentum dependence is key***

*form factor toolbox:*
*BK, Ripken, Saueressig*
*1907.02903*

# Scalar-scalar scattering



$$\mathcal{A}_{\mathfrak{s}}^{\phi\chi} = \frac{4\pi}{3}\Bigg[ -\left(1+\mathfrak{s} f_{Ric\phi\phi}(\mathfrak{s}, m_\phi^2, m_\phi^2)\right)\left(1+\mathfrak{s} f_{Ric\chi\chi}(\mathfrak{s}, m_\chi^2, m_\chi^2)\right) G_C(\mathfrak{s}) \left\{\mathfrak{t}^2 - 4\mathfrak{t}\mathfrak{u} + \mathfrak{u}^2 + 2\left(m_\phi^2 - m_\chi^2\right)^2\right\}$$

$$+ \left((\mathfrak{s}+2m_\phi^2)(1+\mathfrak{s} f_{Ric\phi\phi}(\mathfrak{s}, m_\phi^2, m_\phi^2)) - 12\mathfrak{s} f_{R\phi\phi}(\mathfrak{s}, m_\phi^2, m_\phi^2)\right)$$

$$\times \left((\mathfrak{s}+2m_\chi^2)(1+\mathfrak{s} f_{Ric\chi\chi}(\mathfrak{s}, m_\chi^2, m_\chi^2)) - 12\mathfrak{s} f_{R\chi\chi}(\mathfrak{s}, m_\chi^2, m_\chi^2)\right) G_R(\mathfrak{s})\Bigg]$$

$$G_X(z) = \frac{G_N}{z(1+f_X(z))}$$

$$p_1^2 = p_2^2 = m_\phi^2$$
$$p_3^2 = p_4^2 = m_\chi^2$$

$$\mathfrak{s} = (p_1+p_2)^2$$
$$\mathfrak{t} = (p_1+p_3)^2$$
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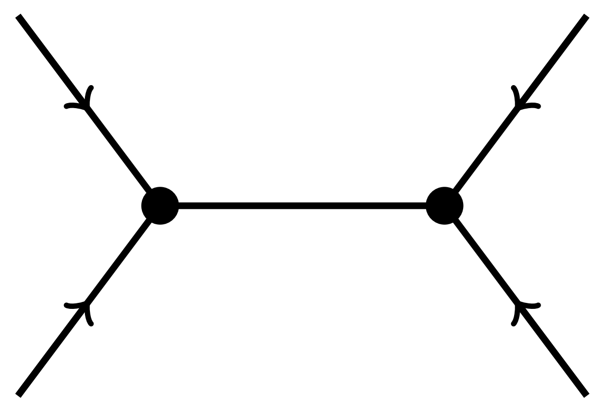
*Draper, BK, Ripken, Saueressig*
*2007.00733, 2007.04396*

# Scalar-scalar scattering



vertex factors · graviton propagator · contraction factor · **spin 2**

$$\mathcal{A}_{\mathfrak{s}}^{\phi\chi} = \frac{4\pi}{3}\Big[ -\left(1+\mathfrak{s} f_{Ric\phi\phi}(\mathfrak{s}, m_\phi^2, m_\phi^2)\right)\left(1+\mathfrak{s} f_{Ric\chi\chi}(\mathfrak{s}, m_\chi^2, m_\chi^2)\right) G_C(\mathfrak{s}) \left\{\mathfrak{t}^2 - 4\mathfrak{t}\mathfrak{u} + \mathfrak{u}^2 + 2\left(m_\phi^2 - m_\chi^2\right)^2\right\}$$
$$+ \left((\mathfrak{s}+2m_\phi^2)(1+\mathfrak{s} f_{Ric\phi\phi}(\mathfrak{s}, m_\phi^2, m_\phi^2)) - 12\mathfrak{s} f_{R\phi\phi}(\mathfrak{s}, m_\phi^2, m_\phi^2)\right)$$
$$\times \left((\mathfrak{s}+2m_\chi^2)(1+\mathfrak{s} f_{Ric\chi\chi}(\mathfrak{s}, m_\chi^2, m_\chi^2)) - 12\mathfrak{s} f_{R\chi\chi}(\mathfrak{s}, m_\chi^2, m_\chi^2)\right) G_R(\mathfrak{s})\Big]$$

**spin 0**

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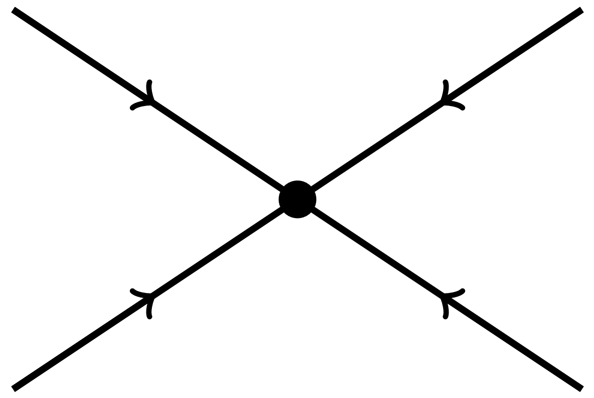

$$\mathcal{A}_4^{\phi\chi} = f_{\phi\chi}\left(\frac{\mathfrak{s}-2m_\phi^2}{2}, \frac{\mathfrak{t}-m_\phi^2-m_\chi^2}{2}, \frac{\mathfrak{u}-m_\phi^2-m_\chi^2}{2}, \frac{\mathfrak{u}-m_\phi^2-m_\chi^2}{2}, \frac{\mathfrak{t}-m_\phi^2-m_\chi^2}{2}, \frac{\mathfrak{s}-2m_\chi^2}{2}\right)$$

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# Beyond scalar-scalar scattering

- similar computations can be done for any scattering event
  - scalar-photon, photon-photon
  - to do: fermions

*BK, Pirlo, Ripken, Saueressig, 2205.01738*
*book chapter: BK, Ripken, Saueressig, 2210.16072*

- using field redefinitions and focussing on **essential** couplings/form factors will be helpful, reduces complexity severely

# Form factors (and the effective action) as a universal language

# Form factors in different approaches

**Stelle gravity**

**Non-local gravity**

**LQG**

**Asymptotic Safety**

**CDT**

# Form factors in different approaches

## Stelle gravity

$$f_{R,C}(\Box) = c_0 + c_1 \ln \Box$$

**recall Luca Buoninfante's talk**

## Non-local gravity

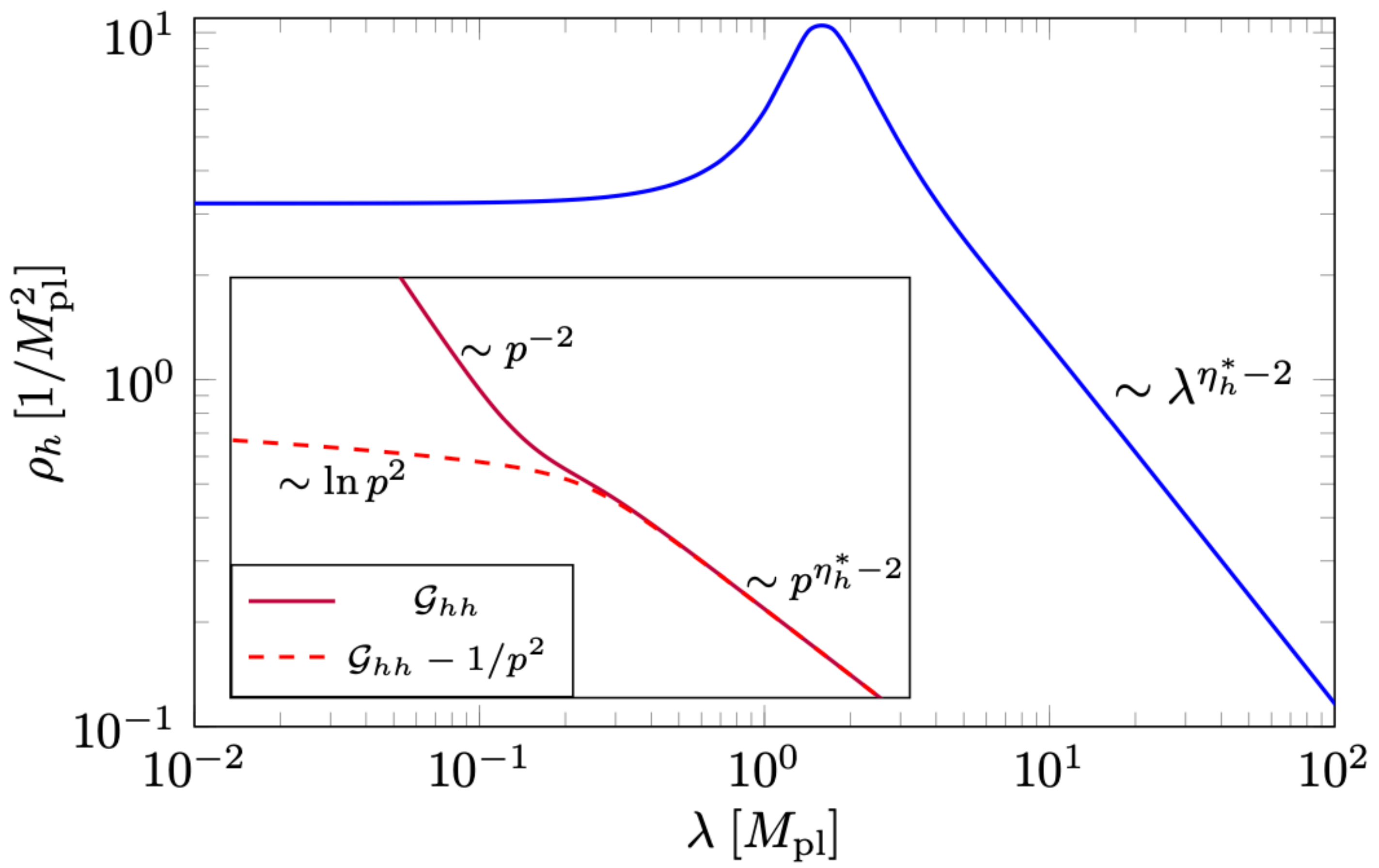
$$f_{R,C}(\Box) \simeq \frac{e^{\Box/M^2} - 1}{\Box}$$

## LQG

$$f_C(\Box) \simeq -\frac{1}{8}\left[\frac{\gamma_+}{m_+^2 + \Box} + \frac{\gamma_-}{m_-^2 + \Box}\right]$$

*Borissova, Dittrich*
*2207.03307*

## Asymptotic Safety




*Fehre, Litim, Pawlowski, Reichert*
*2111.13232*

## CDT

$$f_R(\Box) \simeq \frac{b}{\Box^2}$$

*BK, Saueressig*
*1804.03846*

# Form Factors in CDT

- use correlation function for which CDT data is available, e.g.

$$\mathfrak{V}_2 = \langle \delta V_3(t) \delta V_3(t + \Delta t) \rangle$$

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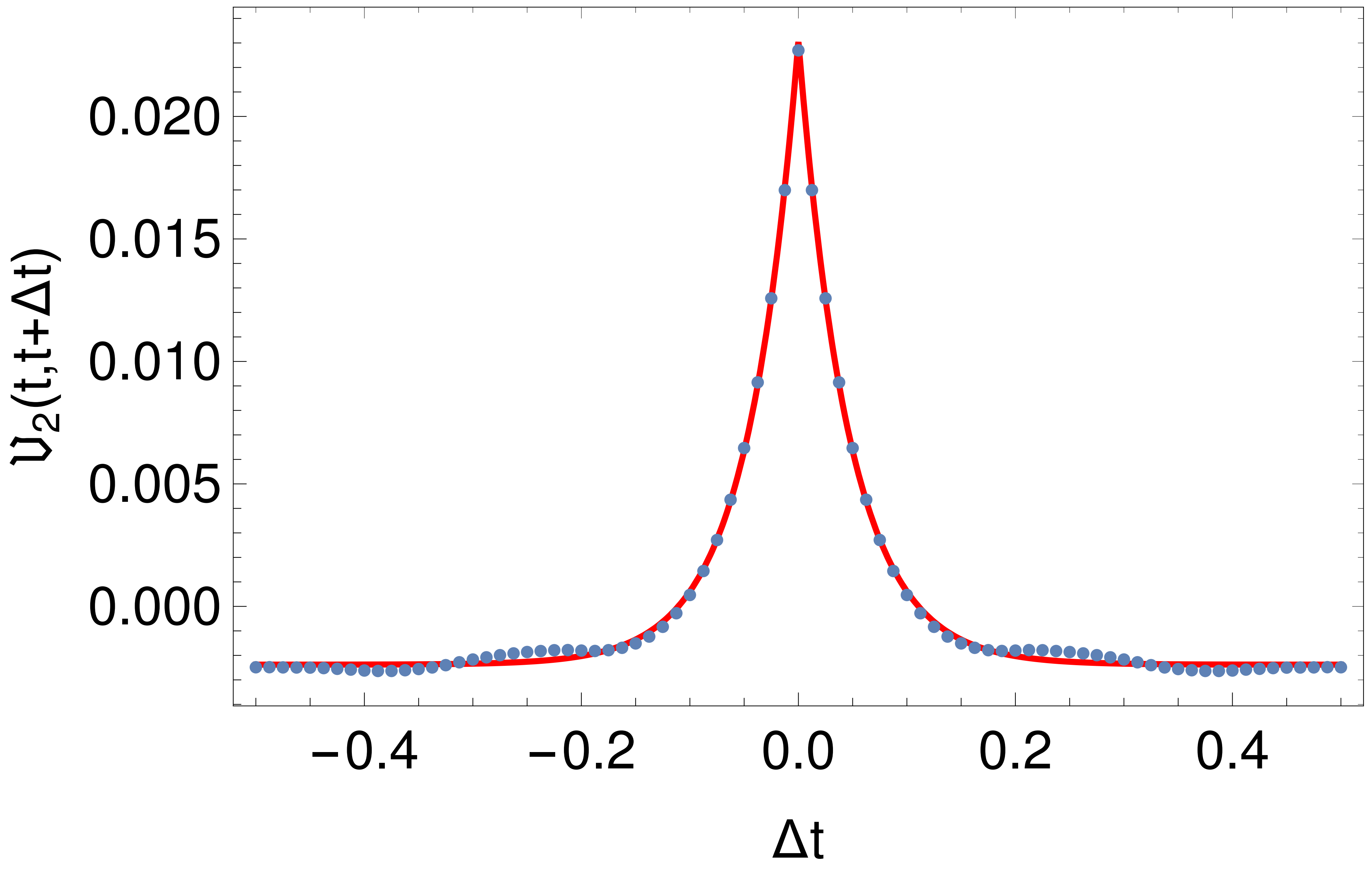


$$f_R(\Box) \simeq \frac{b}{\Box^2}$$

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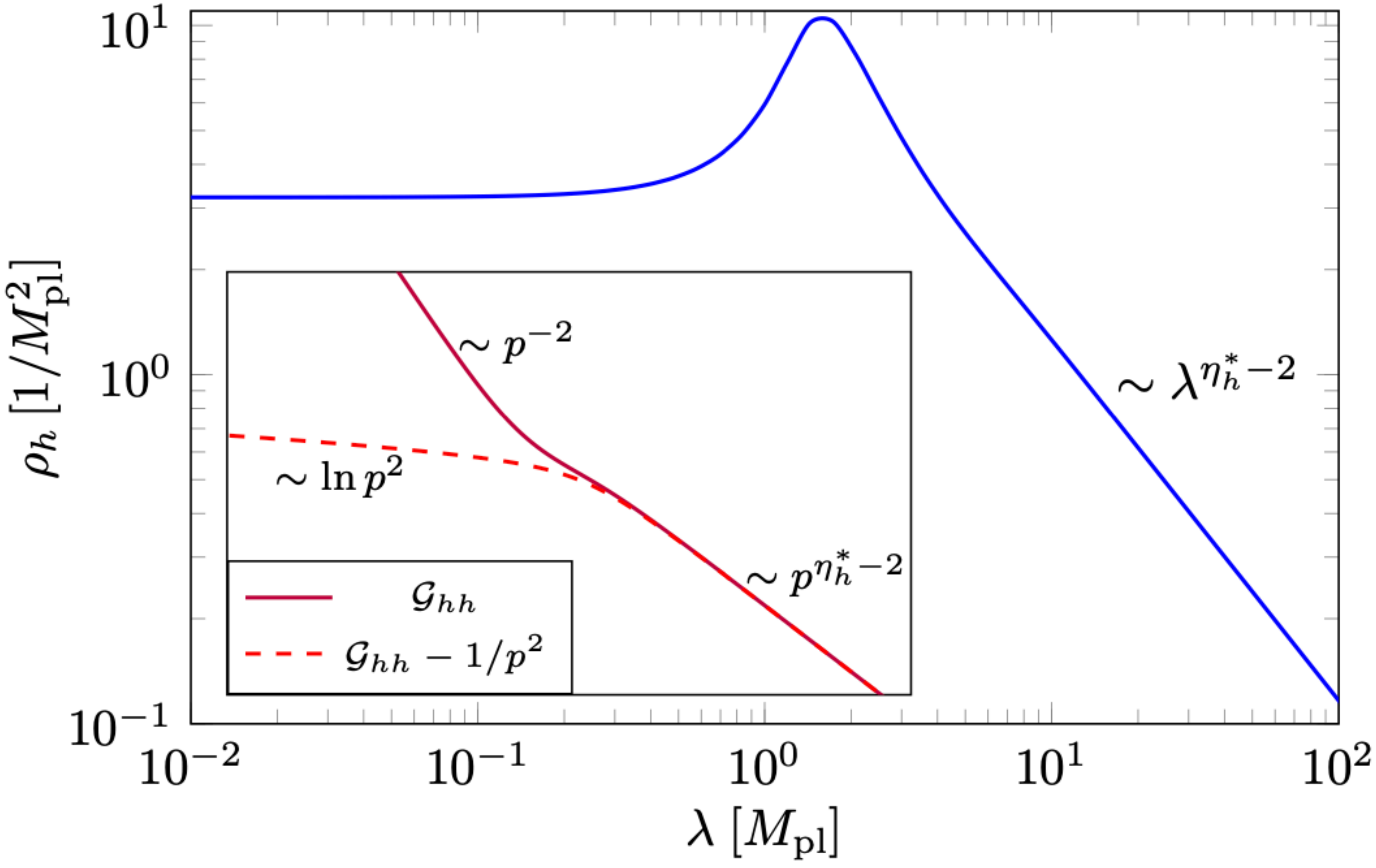
# Graviton spectral function

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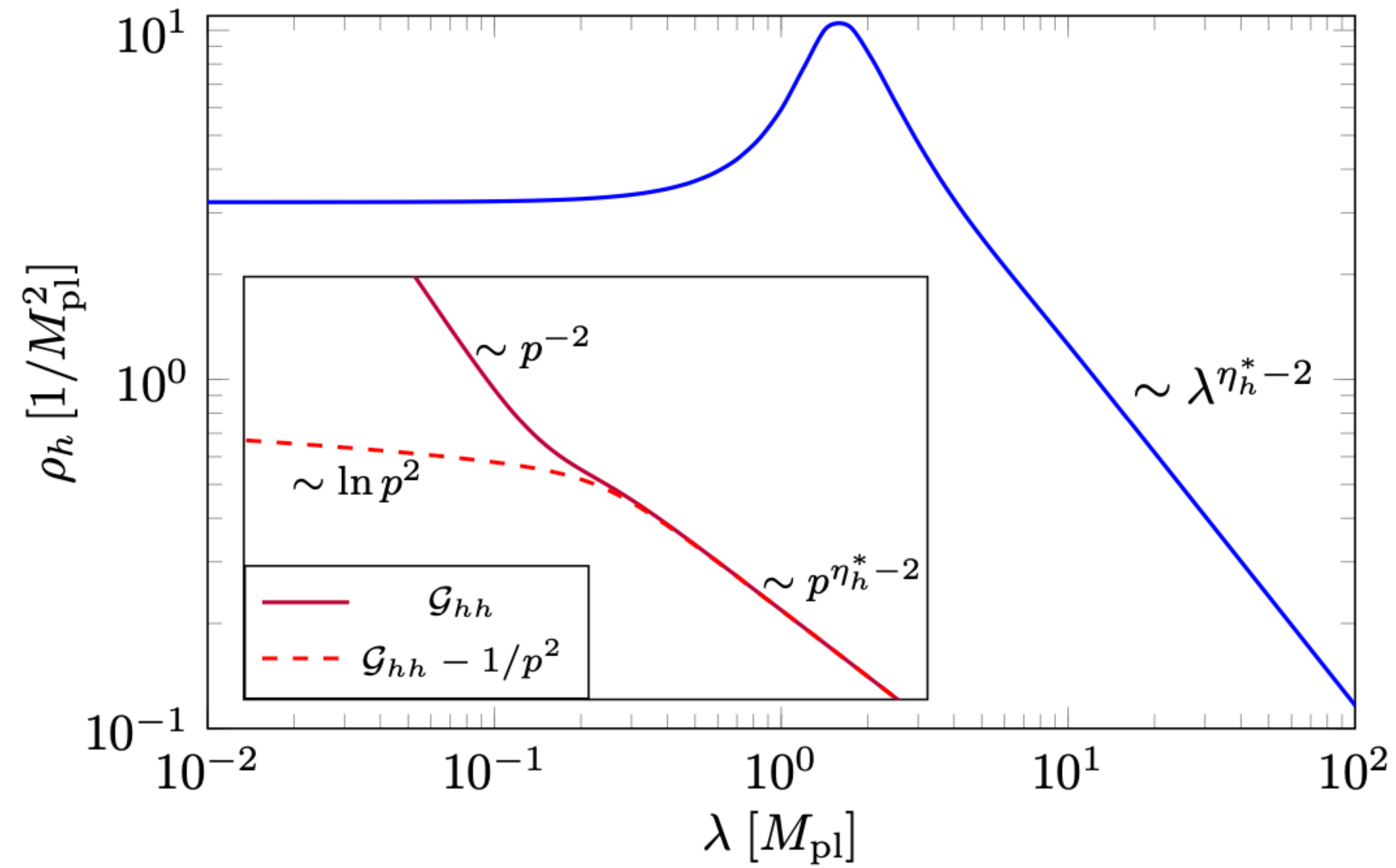
# Graviton spectral function




**Lorentzian computation!**
**Matches EFT in IR!**

*Fehre, Litim, Pawlowski, Reichert*
*2111.13232*

# Comparison of amplitudes

**scalar toy model**

**Asymptotic Safety**

**Non-local gravity**

*BK, Ripken, Saueressig*
*2111.12365*

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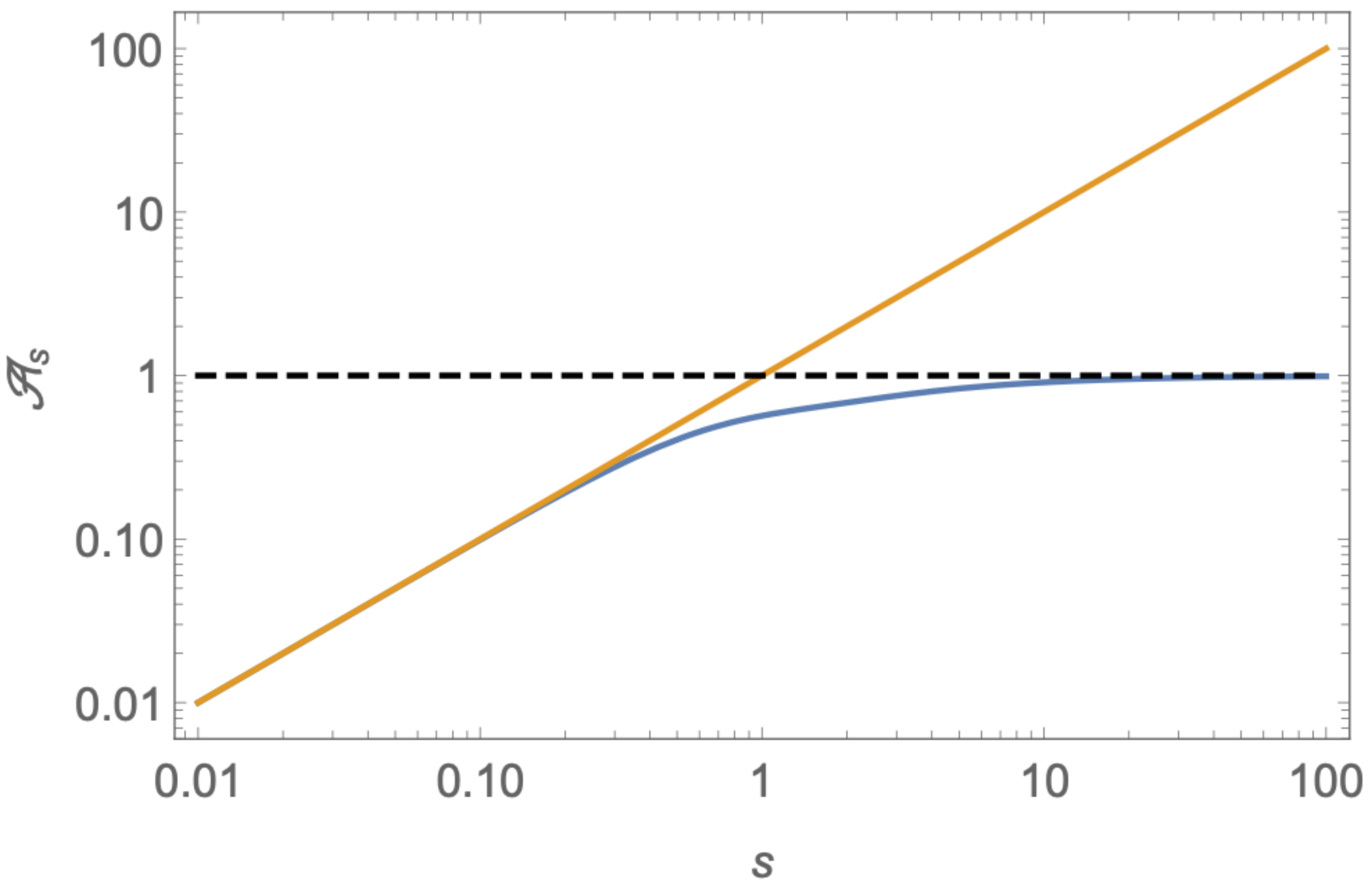
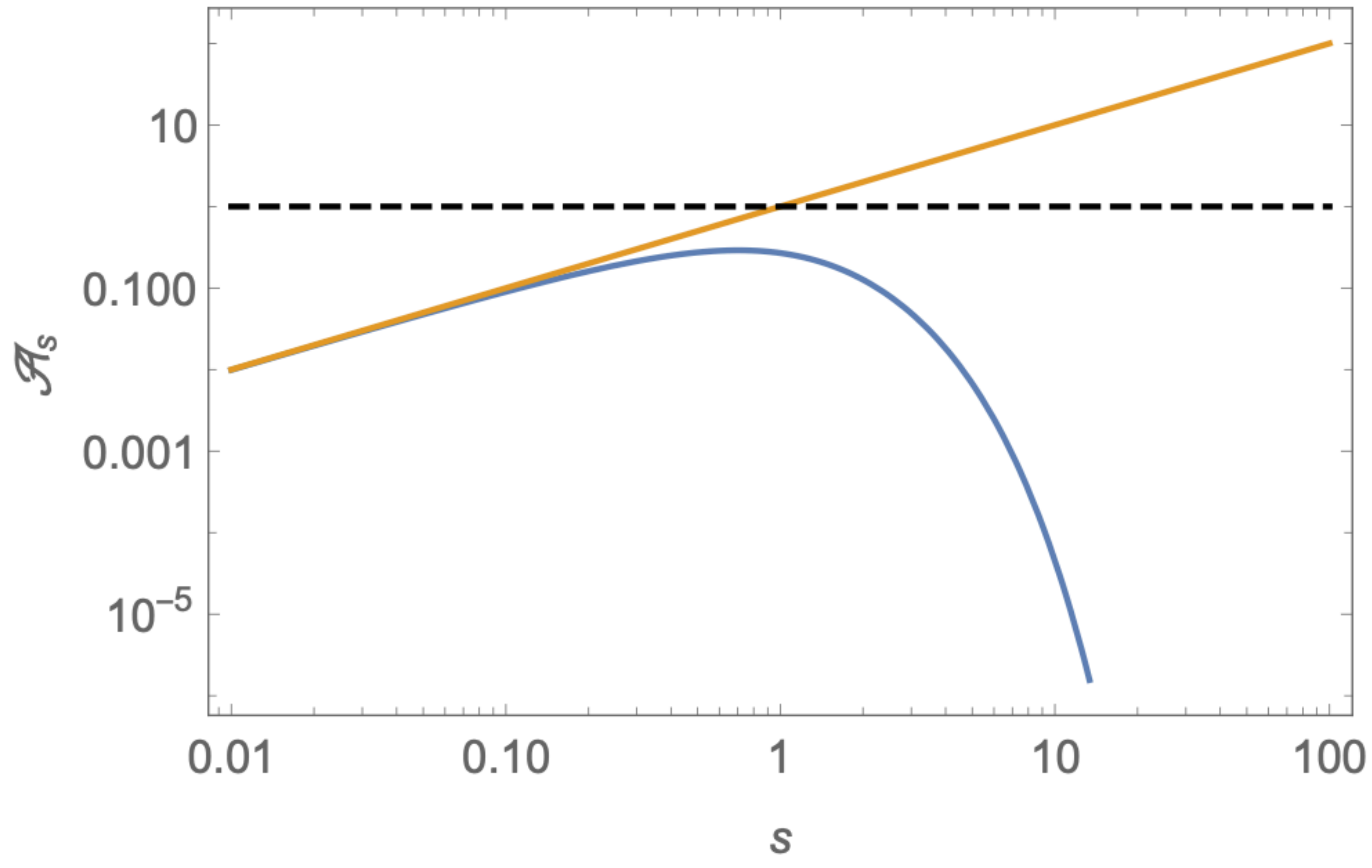
**scalar toy model**




**Asymptotic Safety**




**Non-local gravity**

*BK, Ripken, Saueressig*
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# Summary

# Summary

- falsifiability is at the heart of science, and it should also be at the heart of quantum gravity research

- scattering amplitudes are a useful way to probe quantum gravity

- ingredients can be computed ab initio, no need to guess

- form factors and the effective action are promising tools to **compute and compare QG predictions**